COALITIONAL GAMES FOR A DISTRIBUTED SIGNAL ENHANCEMENT APPLICATION

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ABSTRACT

We consider a scenario in which a number of sensor nodes monitor an area, where several sources are active. Each node has an interest to estimate the signal of a particular source using measurements that, unavoidably, are mixtures of the source signals. Nodes could improve the quality of the signal of interest if they were able to use the signals measured by other nodes, however, in a such a case, communication costs must be properly taken into account. To this end, coalitional game theory is used in our study. In the case where the communication cost is zero, we prove that the cooperation of all nodes is beneficial for all. In contrast, when the communication costs are taken into account, we employ a distributed merge-split coalition formation algorithm to organize the nodes into stable cooperative groups. Simulation results are in accordance with the theoretical findings.

Index Terms— Blind source separation, coalitional game theory, coalition formation algorithms, distributed processing, NTU games.

1. INTRODUCTION

With the continuous decrease in the cost of electronics, ubiquitous miniaturized computing devices equipped with various sensors and communication capabilities have emerged. Making such devices able to self-organize and cooperate, constitutes a major challenge that will offer the potential to significantly improve the quality of the services provided. In this setting, we focus on a case where the objective is to organize the nodes in cooperative groups in order to separate and/or enhance the signals of the sources that are active in an area of interest.

The examined signal enhancement scenario is in close relation to the problem of blindly separating mixtures of independent signals, [1]. More recently, the literature has also focused on various aspects regarding the distributed acquisition and processing involved in the blind source separation problem [2–5]. We consider these works as the closest to our study. In this work, we assume that the blind source separation problem has been solved - e.g., using one of the approaches mentioned above - and the associated mixing model is thus known by the nodes of the network. We turn our attention to the subsequent signal enhancement operation of the network, and consider that each node is a rational and selfish agent that pursues to maximize its benefit. However, due to communication or other costs, each node chooses properly to use the measurements of only a subset of nodes. Also, this node will only send its measurements to nodes with whom it collaborates sharing data. In this setting, we use *coalitional game theory* [6] to construct such sets of nodes and to study when such *coalitions* are stable.

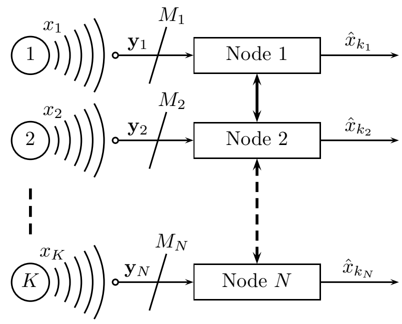

Fig. 1. K sources emit signals $x_1, \dots, x_K$, respectively. Node n uses M_n sensors to measure mixtures of the source signals. Nodes form coalitions, exchange measurements, and each node n provides an estimate of its source of interest x_{k_n}.

Game theory has been applied to a wide range of disciplines such as economics, political sciences, philosophy and more recently, to engineering [7]. In general, game theory can be defined as the study of mathematical models of conflict and cooperation between intelligent rational decision-makers. Unlike non-cooperative game theory where the modelling unit is a single player, coalitional game theory, that is the focus of this work, seeks for optimal coalition structures of players in order to optimize the worth of each coalition. In

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accordance with [6], coalitional games can be categorized as canonical coalitional games, coalition formation games, and coalitional graph games. In this work, the focus is put on the first two categories. To the best of the authors knowledge, this work is the first to address the problem of a distributed signal enhancement from a coalitional game-theoretical aspect.

The rest of the paper is organized as follows: Section 2 provides the formulation of the problem at hand. Section 3 defines and analyses two coalitional games, one with no communication costs and one where communication costs are taken into account. Next, Section 4 describes a coalition formation algorithm for the examined games. Finally, Section 5 presents numerical results that support the theoretical findings, and then the work is concluded.

2. PROBLEM FORMULATION

As demonstrated in Fig. 1, K sources are active at some place of interest. They emit the signals $x_1(t), x_2(t), \ldots, x_K(t)$. We assume that these signals are mutually independent real valued random variables with zero mean. For reasons that will become more clear in the sequel, we also assume that each one of the random variables that model the sources has variance equal to one. Let also $x_1, x_2, \ldots x_K$ denote instances of the respective random variables. Consider also that, in the same area, N nodes monitor the sources, where node n uses M_n sensors (for example, microphones for audio sources) and in general $M_n \geq 1$. Let also the set $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of all the nodes. The measurements of node n are denoted by the vector $\mathbf{y}_n \in \mathbb{R}^{M_n \times 1}$. We assume that the measurements are given by the linear model

$$\mathbf{y}_n = \mathbf{A}_n \mathbf{x} + \mathbf{w}_n \tag{1}$$

where $\mathbf{x} = [x_1 x_2 \cdots x_K]^{\mathrm{T}}$, $\mathbf{A}_n \in \mathbb{R}^{M_n \times K}$ is a matrix that models the attenuation of the source signals and $\mathbf{w}_n \in \mathbb{R}^{M_n \times 1}$ denotes zero mean additive white noise with $\mathbf{w}_n \sim \mathbb{N}(\mathbf{0}, \mathbf{\Sigma}_n)$. We consider also the model that gives the measurements of all nodes if we stack all vectors $\mathbf{y}_n$ into vector

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w} \tag{2}$$

where $\mathbf{y} \in \mathbb{R}^{M \times 1}$, $M = \sum_{n=1}^{N} M_n$ is the number of all measurements, $\mathbf{A} \in \mathbb{R}^{M \times K}$ is constructed by stacking the matrices $\mathbf{A}_n$ and similarly $\mathbf{w} \in \mathbb{R}^{M \times 1}$ is constructed by stacking the respective noise vectors $\mathbf{w}_n$ on top of each other.

We assume that the so-called *mixing matrix* $\mathbf{A}$ in (2) is known. In practical cases, matrix $\mathbf{A}$ can be estimated using a Blind Source Separation (BSS) algorithm [1]. Since most BSS algorithms require the number of measurements to be at least equal to the number of sources, $M \geq K$ should hold for the computation of matrix $\mathbf{A}$. Also, due to the inherent scale ambiguity in BSS algorithms, we assume that all sources have unit variance, which is a common assumption in BSS literature.

Consider now that node n is interested in the estimation of source $k_n \in \{1, 2, \ldots, K\}$ but, due to communication or other costs, it can only utilize a subset of the measured signals, given by

$$\mathbf{y}_{\mathcal{S}} = \mathbf{A}_{\mathcal{S}} \mathbf{x} + \mathbf{w}_{\mathcal{S}} \ , \tag{3}$$

where $\mathbf{A}_{\mathcal{S}}$ (respectively $\mathbf{w}_{\mathcal{S}}$) is generated from $\mathbf{A}$ (respectively $\mathbf{w}$) if we keep only the $\mathbf{A}_i$ (respectively $\mathbf{w}_i$) with $i \in \mathcal{S} \subseteq \mathcal{N}$. The set $\mathcal{S}$ denotes the *cooperating nodes* that exchange their measurements to aid each other in the signal enhancement task. Our scope is to study the conditions under which it is beneficial for the nodes to take part in such a coalition.

3. SIGNAL ENHANCEMENT GAMES

Coalitional games are also categorized to (a) Transferable Utility (TU) games and (b) Non Transferable Utility (NTU) games [7]. In TU games, a real number is used to measure the utility of any subset of players. In NTU games, examined here, the value of a coalition is a vector used to assign each member of the coalition its own utility.

3.1. A Game with no Communication Costs

Let us define an NTU coalitional game $(\mathcal{N}, v)$, where the set $\mathcal{N}$ is the set of players (nodes) and $v(\mathcal{S}) \subseteq \mathbb{R}^{|\mathcal{S}|}$ contains the payoff vectors achievable by the players in coalition $\mathcal{S}$. In particular, if $\mathbf{p} \in v(\mathcal{S})$ is a payoff vector, then the payoff of player $n \in \mathcal{S}$ is an element of $\mathbf{p}$ that we define as

$$p_n(\mathcal{S}) = -E\left[(x_{k_n} - \hat{x}_{k_n,\mathcal{S}})^2\right] \ . \tag{4}$$

That is, the payoff of player n in $\mathcal{S}$ is the negative of the Mean Squared Error (MSE) achieved for the estimation of the source of interest x_{k_n}. Each player is rationally selfish and attempts to maximize its own payoff, or equivalently to minimize its MSE. Taking into account the linear model of (3), the estimate $\hat{x}_{k_n,\mathcal{S}}$ is defined as

$$\hat{x}_{k_n,\mathcal{S}} = \mathbf{b}_{k_n,\mathcal{S}}^{\mathrm{T}} \mathbf{y}_{\mathcal{S}} \ , \tag{5}$$

i.e., as the output of a linear filter $\mathbf{b}_{k_n,\mathcal{S}}$ that acts on the available data $\mathbf{y}_{\mathcal{S}}$. The choice of the filter coefficients corresponds to the action/strategy adopted by the respective player.

3.1.1. Analysis of the Game with no Communication Costs

Let us first characterize the sets $v(\mathcal{S})$ for $\mathcal{S} \subseteq \mathcal{N}$. According to [8], the optimal filter $\mathbf{b}_{k_n,\mathcal{S}}$ is given by the equation

$$\mathbf{b}_{k_n,\mathcal{S}}^{(\mathrm{OPT})} = \mathbf{R}_{\mathbf{y}_{\mathcal{S}},\mathbf{y}_{\mathcal{S}}}^{-1} \mathbf{r}_{\mathbf{y}_{\mathcal{S}},x_{k_n}} \tag{6}$$

where $\mathbf{R}_{\mathbf{y}_{\mathcal{S}},\mathbf{y}_{\mathcal{S}}} = E\left[\mathbf{y}_{\mathcal{S}} \mathbf{y}_{\mathcal{S}}^{\mathrm{T}}\right]$ and $\mathbf{r}_{\mathbf{y}_{\mathcal{S}},x_{k_n}} = E\left[\mathbf{y}_{\mathcal{S}} x_{k_n}\right]$ are the auto-correlation matrix of the input signal and the cross-correlation vector between the input vector and the desired output source, respectively. Now, using equation (3), it is easy to verify that

$$\mathbf{b}^{(\text{OPT})}_{k_n,\mathcal{S}} = \left(\mathbf{A}_{\mathcal{S}}\mathbf{A}^{\text{T}}_{\mathcal{S}} + \mathbf{\Sigma}_{\mathcal{S}}\right)^{-1} \mathbf{A}_{\mathcal{S}}\mathbf{d}_{k_n} \ , \tag{7}$$

where $\mathbf{\Sigma}_{\mathcal{S}}$ is the covariance matrix of the noise at the sensors of the nodes in the set $\mathcal{S}$ and $\mathbf{d}_{k_n} \in \mathbb{R}^{K\times 1}$ is a vector with all its elements equal to zero, except for the k_n-th element that is equal to one. The respective Minimum MSE (MMSE) for the estimation of source k_n using the measurements from the nodes in $\mathcal{S}$, is given by

$$\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}} = 1 - \mathbf{d}^{\text{T}}_{k_n}\mathbf{A}^{\text{T}}_{\mathcal{S}}\left(\mathbf{A}_{\mathcal{S}}\mathbf{A}^{\text{T}}_{\mathcal{S}} + \mathbf{\Sigma}_{\mathcal{S}}\right)^{-1} \mathbf{A}_{\mathcal{S}}\mathbf{d}_{k_n} \ , \tag{8}$$

where we have used the assumption that all sources have unit variance and zero mean. We can now show the following:

Lemma 1. *The set $v(\mathcal{S})$ for the Signal Enhancement Game, with no communication costs, is comprehensive.*

Proof. The set $v(\mathcal{S})$ is called *comprehensive*, if it holds that when $\mathbf{p} \in v(\mathcal{S})$ and $\mathbf{q} \in \mathbb{R}^{|\mathcal{S}|}$ are such that $\mathbf{q} \leq \mathbf{p}$, then $\mathbf{q} \in v(\mathcal{S})$, [6]. In other words, the players can achieve any payoff that is smaller or equal to any achievable payoff. It is easy to verify that the equation

$$E\left[(x_{k_n} - \mathbf{b}^{\text{T}}_{k_n,\mathcal{S}}\mathbf{y}_{\mathcal{S}})^2\right] = C \tag{9}$$

always has at least one solution with respect to $\mathbf{b}_{k_n,\mathcal{S}}$, as long as the constant $C \geq \text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}}$. Thus, it follows that all payoffs smaller that the optimal one are achievable. □

Based on the above, it is easy to verify that the set of payoff vectors is equal to

$$v(\mathcal{S}) = \left\{\mathbf{p} \in \mathbb{R}^{|S|} : \forall n \in S, p_n(\mathcal{S}) \leq -\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}}\right\} \ , \tag{10}$$

where $p_n(\mathcal{S})$ denotes the n-th element of the payoff vector $\mathbf{p}$. From the above relation, it is easy to note that the set $v(\mathcal{S})$ is a so-called box, where $-\infty < p_n \leq -\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}}$. We now proceed with the following:

Lemma 2. *The examined Signal Enhancement Game, with no communication costs, is canonical.*

Proof. According to [6], a cooperative game is termed as *canonical* if it fulfills two requirements: (a) The value of a coalition $\mathcal{S}$ is determined by the members of $\mathcal{S}$ and not, in any sense, by the players in $\mathcal{N} \setminus \mathcal{S}$, and (b) The game is *superadditive*, a property that is formally defined as

$$\begin{aligned} v(\mathcal{S}_1 \cup \mathcal{S}_2) \quad \supseteq \quad & \{\mathbf{p} \in \mathbb{R}^{|\mathcal{S}_1\cup\mathcal{S}_2|} : (p_n)_{n\in\mathcal{S}_1} \in v(\mathcal{S}_1), \\ & (p_{n'})_{n'\in\mathcal{S}_2} \in v(\mathcal{S}_2)\} \\ & \forall \mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{N} \quad \text{s.t.} \quad \mathcal{S}_1 \cap \mathcal{S}_1 = \emptyset \end{aligned} \tag{11}$$

It is obvious that our examined game, by definition, fulfills requirement (a). Such games are termed as being *in characteristic form*. In order to prove that the examined game is furthermore superadditive, it suffices to show that

$$\forall n \in \mathcal{S}_1, \text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}_1\cup\mathcal{S}_2} \leq \text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}_1} \tag{12}$$

and

$$\forall n' \in \mathcal{S}_2, \text{MSE}^{(\text{OPT})}_{k_{n'},\mathcal{S}_1\cup\mathcal{S}_2} \leq \text{MSE}^{(\text{OPT})}_{k_{n'},\mathcal{S}_2} \ . \tag{13}$$

Since in the coalition $\mathcal{S}_1 \cup \mathcal{S}_2$ all nodes utilize an augmented measurement vector $\mathbf{y}_{\mathcal{S}_1\cup\mathcal{S}_2}$ and the optimal (in the MMSE sense) filter is computed using this augmented measurement vector, then it follows that the above inequalities do hold true. In other words, in the coalition $\mathcal{S}_1 \cup \mathcal{S}_2$, the players can always revert back to their previous behavior but they also have the opportunity to utilize more information, and thus decrease their MSE. □

Due to the fact that, in a canonical game, cooperation is never harmful with respect to the non-cooperative case, it is important to study the properties of the grand coalition, i.e., the coalition of all nodes. To this end, we will employ the, probably most studied, solution concept, known as the *core* [7]. The core of a canonical game is the set of payoff vectors for which it holds that no coalition $\mathcal{S} \subset \mathcal{N}, \mathcal{S} \neq \emptyset$ has an incentive to split off. In order to solve an NTU game using the core, the value v of the game is often assumed to satisfy the following three requirements, for any coalition $\mathcal{S}$ [6]:

1. $v(\mathcal{S})$ must be a closed and convex subset of $\mathbb{R}^{|\mathcal{S}|}$
2. $v(\mathcal{S})$ must be comprehensive
3. The set $\{\mathbf{p} : \mathbf{p} \in v(\mathcal{S}) \quad \text{and} \quad p_n \geq z_n, \forall n \in \mathcal{S}\}$, where $z_n = \max\{p_n : \mathbf{p} \in v(\{n\})\} < \infty \ \forall n \in \mathcal{N}$, must be a bounded subset of $\mathbb{R}^{|\mathcal{S}|}$.

If a canonical NTU game has a v that satisfies the above, then the core of the game can be defined as

$$\mathcal{C}_{\text{NTU}} = \{\mathbf{p} \in v(\mathcal{N}) : \forall \mathcal{S}, \nexists \mathbf{q} \in v(\mathcal{S}), \text{s.t. } q_n > p_n, \forall n \in \mathcal{S}\} \tag{14}$$

Theorem 1. *The examined Signal Enhancement Game, with no communication costs, has a non-empty core.*

Proof. We will first show that the three above requirements are fulfilled. First, from equation (10), it is easy to verify that $v(\mathcal{S})$ is a closed and convex subset of $\mathbb{R}^{|\mathcal{S}|}$. Second, the set $v(\mathcal{S})$ is comprehensive according to Lemma 1. For the third requirement, we first note that the maximum utility z_n of node n when not cooperating is $z_n = -\text{MSE}^{(\text{OPT})}_{k_n,\{n\}}$. Thus, the sets involved in requirement 3 will be given by

$$\left\{\mathbf{p} \in \mathbb{R}^{|\mathcal{S}|} : \forall n \in \mathcal{S}, -\text{MSE}^{(\text{OPT})}_{k_n,\{n\}} \leq p_n \leq -\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}}\right\} \tag{15}$$

which is a bounded subset (box) of $\mathbb{R}^{|\mathcal{S}|}$. Of course, from Lemma 2, we know that $-\text{MSE}^{(\text{OPT})}_{k_n,\{n\}} \leq -\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{S}}$.

Since all three requirements are satisfied, the definition of the core in (14) holds. Furthermore, if we consider the payoff vector $\mathbf{p}_o \in \mathbb{R}^N$ with elements $p_{o,n} = -\text{MSE}^{(\text{OPT})}_{k_n,\mathcal{N}}$, it can be shown that it lies in the core, and thus, the core is non-empty. □

3.2. A Game with Communication costs

We now attempt to study the case in which the nodes of the network have some communication costs. In particular, we define an NTU coalitional game $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players/nodes and $v(\mathcal{S}) \subseteq \mathbb{R}^{|\mathcal{S}|}$ is the set of payoff vectors for a coalition $\mathcal{S} \subseteq \mathcal{N}$. If $\mathbf{p} \in v(\mathcal{S})$ is a payoff vector, then the payoff of player $n \in \mathcal{S}$ is an element of $\mathbf{p}$, given as

$$p_n(\mathcal{S}) = -E\left[(x_{k_n} - \hat{x}_{k_n,\mathcal{S}})^2\right] - C_{n,\mathcal{S}} \ , \qquad (16)$$

where $C_{n,\mathcal{S}}$ stands for the total communication costs required by node n when it is a member of coalition $\mathcal{S}$. We consider the case in which communication among the nodes of the network is based only on one-hop links. In other words, nodes do not store and forward packets. The utility in (16) can represent two separate cases, and in particular (a) the case in which for any coalition $\mathcal{S}$, each node $n \in \mathcal{S}$ transmits its measurements to all other nodes in $\mathcal{S}$ using $|\mathcal{S}| - 1$ direct (point-to-point) links (i.e., complete graph) and (b) the case in which it uses a broadcast message with enough power to reach the furthest node in $\mathcal{S}$.

3.2.1. Analysis of the Game with Communication Costs

Theorem 2. *The examined Signal Enhancement Game, with communication costs included, has, in general, empty core.*

Proof. It is easy to prove that in general the core is empty, if one considers high communication costs. In fact, we can force any coalition to split if we increase enough one or more communication costs. □

Since the nodes will not in general form the grand coalition, it is of interest to apply a coalition formation algorithm and analyze the properties of the coalitional structure being formed. Toward this goal, our focus is put on the so-called merge-split approach, initially presented in [9], and applied in the context of several applications [6], [10]- [13].

4. A COALITION FORMATION ALGORITHM

In order to derive a coalition formation algorithm for the examined game, we proceed with some definitions. A *collection* of coalitions is a set of mutually disjoint coalitions, i.e., $\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\}$, where $\mathcal{S}_l \subset \mathcal{N}$ for $l = 1, 2, \ldots, L$. If a collection $\mathcal{P}$ covers all the nodes in $\mathcal{N}$, then the collection $\mathcal{P}$ is also a *partition* of $\mathcal{N}$. The *Pareto order* operator $\triangleright$ for comparing two collections $\mathcal{P} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\}$ and $\mathcal{Q} = \{\mathcal{S}'_1, \mathcal{S}'_2, \ldots, \mathcal{S}'_{L'}\}$, that are partitions of the same set $\mathcal{A} \subseteq \mathcal{N}$, is defined as

$$\mathcal{P} \triangleright \mathcal{Q} \longleftrightarrow p_n(\mathcal{P}) \geq p_n(\mathcal{Q}), \qquad \forall\, n \in \mathcal{A} \qquad (17)$$

with at least one node satisfying the strict inequality.

We can now define an algorithm for coalition formation that is based on two simple steps, i.e., merge and split [9], [10], as follows:

- Merge any collection of coalitions $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\}$ into a single coalition, if the merged coalition is "greater" in terms of the Pareto order, i.e. if $\bigcup_{l=1}^{L} \mathcal{S}_l \triangleright \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\}$
- Split any coalition into a collection of smaller coalitions $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\}$, if that collection is "greater" in terms of the Pareto order, i.e., if $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_L\} \triangleright \bigcup_{l=1}^{L} \mathcal{S}_l$.

Note that coalitions will merge (or split) only if there is at least one node that is improving its payoff while there is no decrease in the payoffs of the other nodes of the coalitions involved in the merge (or split) operation. The algorithm terminates for any initialization, and the coalition structure always converges to a $\mathbb{D}_{hp}-$stable coalition structure [9], i.e., no group of nodes has an incentive to perform a merge or a split operation. The merge-split protocol can be practically implemented in several different ways, with respective trade-offs between the required complexity and the payoffs of the achieved solution. A possible implementation that was followed in our experiments is described in next section.

To cope with possible node mobility, time varying mixing matrix and/or noise variances, the coalition formation process may be repeated periodically during the network operation.

5. SIMULATION RESULTS

In this section, we aim to verify the theoretical discussion and the effectiveness of the merge-split algorithm in the framework of an acoustics illustrative example. We assume $K = 3$ sound sources, and $N = 7$ nodes where each of them is interested in a single source. For simplicity, it is assumed that each node possesses a single microphone, i.e., $M_n = 1$, for $n \in \{1, \ldots, 7\}$. The network has been randomly generated to lie withing the unit square (see Fig. 2). In other words, the coordinates of both nodes and sound sources are drawn from a uniform distribution over the unit square. Each source influences all nodes, and this is modeled using a distance-based mixing matrix. Specifically, the (n, k)-th element of matrix A is defined as $a_{n,k} = 1/(d_{n,k}/d_0)^2$, where $d_0 = 0.2$ is a reference distance, with $d_{n,k}$ being the distance between a node n and a source k. Furthermore, we assume that each node is interested in the source that is the closest to it, while the noise variance of each node is chosen from the interval $(0.01, 0.36)$. The communication cost between the nodes n and n' is modeled with a simple exponential model, i.e., $C_{n,n'} = \beta \cdot e^{(d_{n,n'}/d_0)}$, where $d_{n,n'}$ denotes now the distance between the nodes n and n', while β is a normalization coefficient. Assuming broadcast communication, the total communication cost for a node n in order to establish a coalition $\mathcal{S}$ is given by $C_{n,\mathcal{S}} = \max_{n' \in \mathcal{S} \setminus \{n\}} \{C_{n,n'}\}$.

The merge-split protocol is practically implemented as follows. Firstly, any coalition T_i from an initial network partition $\mathcal{T}$ starts the merging process by performing pairwise negotiations with other coalitions. In case that a merge occurs, the newly formed coalition continues the search for merging

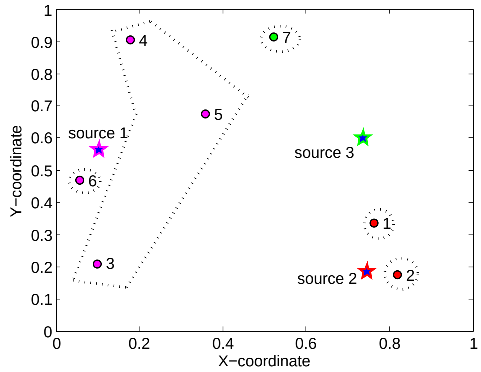

Fig. 2. Snapshot of the coalitional structure (dot lines) using the merge-split algorithm, for $\beta = 0.001$ with 7 nodes (circles) and 3 audio sources (stars) deployed in the unit square.

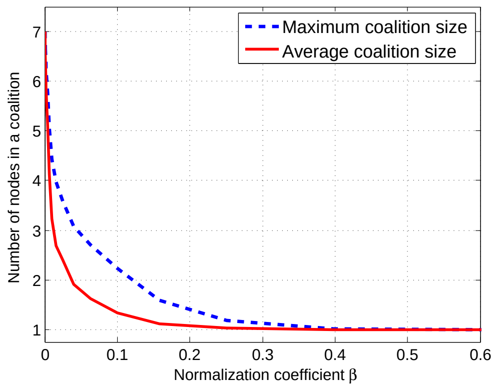

Fig. 3. Maximum and average coalition size as a function of the communication cost.

until it is possible. Next, the merging process is repeated for all remaining coalitions from $\mathcal{T}$ that have not been merged yet. In the second step, the resulting coalition(s) are performing split operations, if any is possible.

Figure 2 shows a coalitional structure snapshot resulting from the merge-split protocol, for a single network realization, for the normalization coefficient β being fixed to 0.001. The stars represent the sources while the nodes are depicted as circles. The color of the nodes indicates the source of their interest, and the dotted lines/ellipses illustrate the coalitions being formed in this scenario.

Next, Figure 3 shows the sizes of coalitions resulting from the merge-split protocol as the normalization coefficient of the communication cost is increased. In particular, the maximum and the average sizes of the coalitions being formed are plotted. The results have been averaged over 100 random network realizations. The figure shows that for zero and relatively small communication costs, the coalition structures corresponds to the grand coalition, i.e., all nodes are exchanging information among themselves. On the other hand, as the communication costs get increased, the coalitions break off, and finally, they reduce to the non-cooperative nodes.

6. CONCLUSION

In this work, we examined a distributed signal enhancement scenario in a coalitional game theoretic framework. In particular, our objective was to study when it is beneficial for the nodes to take part into coalitions that help them decrease their MSE. In the case of zero communication costs, it was shown that the coalition of all the nodes is formed. When communication cost is non negligible, a merge-split algorithm can be employed to construct stable coalitions. Numerical results were provided that support the theoretical findings.

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