# Turbo Equalization of Non-Linear Satellite Channels using Soft Interference Cancellation

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***Abstract*— Satellite communication channels can be well described as non-linear functions with memory. The Volterra series representation provides accurate modeling of satellite channel dynamics, and thus, it constitutes a widely used approach to mathematically describe them. In this work, iterative correction of the non-linear distortion introduced by such channels is considered, by employing a soft interference canceller operating in a turbo equalization framework.**

## I. Introduction

Inspired by the advent of turbo codes [1], turbo equalization (TE) [2] has emerged as a promising technique for drastic reduction of intersymbol interference (ISI) in frequency selective wireless channels. Most of the research effort on TE has focused on means for reducing the computational complexity of the involved soft-input soft-output (SISO) equalization algorithm, as compared to the complexity of the trellis diagram based equalizer of [2]. In particular, in [3], a soft interference canceller (SIC) was presented, whose filters were adaptively optimized with respect to a mean squared error criterion. In [4], several equalization algorithms of different computational complexity were proposed. More specifically, appropriate minimum mean square error (MMSE) criteria were defined in [4], in which the involved expectations were expressed with respect to both the probability density function of the noise and the a-priori probabilities about the transmitted symbols. In [5], optimal in the MMSE sense transfer functions were derived for the equalizer filters and it was shown that under certain conditions, the proposed equalizer is equivalent to one previously proposed in [4]. In [6], it was shown that the performance of the equalizer proposed in [4] can be obtained by keeping the equalizer filters constant, while changing their inputs. Based on this property, a low complexity scheme has been developed.

It is really noticeable that although the linear turbo equalization problem has been extensively studied, very few researchers have considered the potential application of iterative turbo techniques in non-linear channels and in satellite channels in particular [7]. Specifically, most of the research effort in mitigating the non-linear effects of satellite channels has mainly focused on the so-called “pre-distortion” and “post-compensation” tecniques [8].

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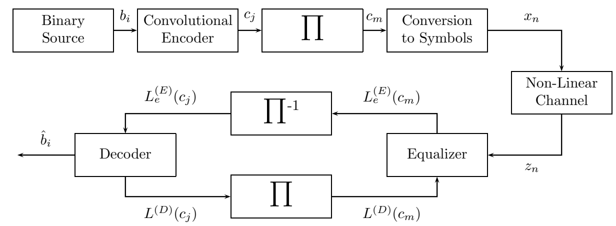


Fig. 1. The transmission model

In this work, we use the Volterra model representation of satellite channels, studied in [9], to derive a non-linear soft interference canceller (NL-SIC) that minimizes the mean squared error (MSE) between the transmitted symbols and the output of the canceller. Minimization is carried out by assuming that past symbols have been detected correctly, and thus, the derived canceller can be viewed as the non-linear extension of the interference canceller described in [3] for the linear channel case. In practice, in place of the required past symbols, their soft estimates are computed using the available a-priori probabilities provided to the SISO equalizer by the channel decoder.

The remaining of this work is organized as follows: In Section II we describe the overall transmission system considered, and give a brief overview of the turbo equalization procedure. In Section III the non-linear satellite channel is studied and the associated Volterra model is introduced. In Section IV, the proposed SISO equalizer is derived. In particular, in three subsections, we focus on the computation of the equalizer filters, the input to these filters and the output of the proposed equalizer respectively. Finally, we present some numerical results and draw our conclusions.

## II. System Model

Consider the transmission model depicted in Fig. 1. A discrete memoryless source generates binary data $b_i, i = 1 \dots S$. These data, in blocks of length $S$, enter a convolutional encoder of rate $R$, so that new blocks of $S/R$ bits ($c_j, j = 1 \dots S/R$) are produced, where $S/R$ is assumed integer and no trellis termination is assumed. The output of the convolutional encoder is then permuted by an interleaver (denoted as $\Pi$ in Fig. 1), so as to form the corresponding block of bits $c_m, m = 1 \dots S/R$. The output of the interleaver forms groups of $q$ bits and each group is mapped into a $2^q$-ary symbol from

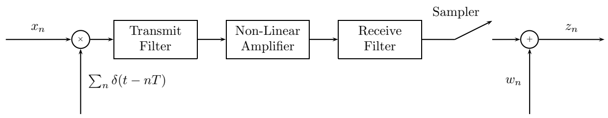


Fig. 2. Discrete input - discrete output model of the non-linear channel

the alphabet $A = \{\alpha_1, \alpha_2, \ldots, \alpha_{2^q}\}$. The symbol alphabet $A$ is assumed to be PSK as in [9]. The resulting symbols $x_n, n = 1 \ldots \frac{S}{Rq}$ are finally transmitted through the wireless channel.

At the receiver, we employ an equalizer to compute soft estimates of the transmitted symbols. As a part of the equalizer is also a scheme that transforms the soft estimates of the symbols into soft estimates of the bits that correspond to those symbols. The output of the equalizer is the log-likelihood ratio $L_e^{(E)}(c_m), m = 1 \ldots S/R$, where the subscript stands for "extrinsic" and the superscript denotes that this log-likelihood ratio comes from the equalizer. The operator $L(\cdot)$ applied to a binary random variable $y$ is defined as

$$L(y) = \ln\left(\frac{Pr(y=1)}{Pr(y=0)}\right).$$

In the sequel, the log-likelihood ratios $L_e^{(E)}(c_m)$ are de-interleaved and enter a soft convolutional decoder, implemented here as a MAP decoder. We stretch the fact that the convolutional decoder operates on the code bits $c_j$ of the code and not on the information bits $b_i$. The log-likelihood ratios $L^{(D)}(c_j)$ at the output of the decoder are first interleaved and then enter the SISO equalizer as a-priori probabilities information. These a-priori probabilities are combined with the output of the channel via a SISO equalization algorithm, which computes new soft estimates about the transmitted bits. This procedure is iterated until a termination criterion is satisfied [10]. Here we choose to use a fixed number of iterations. At the last iteration, the decoder operates on the information bits $b_i$ and delivers the hard estimates $\hat{b}_i$.

## III. The Non-Linear Satellite Channel

Fig. 2 depicts a discrete time model for the non-linear satellite channel studied in [9]. In particular, the sequence of PSK symbols is transformed into a continuous time signal after multiplication with the comb function $\sum_n \delta(t-nT)$, with $T$ denoting the symbol period. Then, the continuous time signal is convolved with the transmit filter, amplified by the non-linear, zero memory power amplifier and processed by the receive filter. As a result, the distortion induced contains both non-linear effects due to the amplifier and intersymbol interference due to the transmit and receive filters. It should be noted that the intersymbol interference could be seriously increased, if the transmission channel also involves multipath propagation. In such a case, the impulse response of the multipath channel should also appear in Fig. 2, as a linear filter between the amplifier and the receive filter. Finally, the output of the transmit filter is sampled and further distorted by zero mean additive white Gaussian noise.

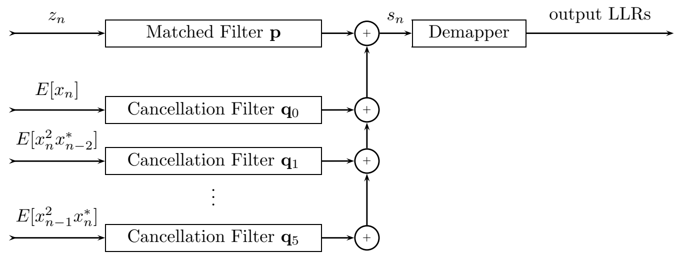


Fig. 3. The non-linear SIC

According to [9], the non-linear satellite channel of Fig. 2 can be described by a Volterra model, in which only odd-order nonlinearities are present. Thus, considering the channel proposed in [9] and keeping for simplicity only up to third order nonlinearities, the output of the channel can be written as

$$\begin{aligned} z_n &= w_n + \sum_{i=0}^{L_0} H_i^{(1)} x_{n-i} + H_{002}^{(3)} x_n^2 x_{n-2}^* + H_{330}^{(3)} x_{n-3}^2 x_n^* \\ &+ H_{001}^{(3)} x_n^2 x_{n-1}^* + H_{003}^{(3)} x_n^2 x_{n-3}^* + H_{110}^{(3)} x_{n-1}^2 x_n^* \qquad (1) \end{aligned}$$

where $w_n$ is zero mean additive white Gaussian noise (AWGN) with variance $\sigma_w^2$, $H_i^{(1)}$ denote the linear channel coefficients and $H_{ijk}^{(3)}$ the third order non-linear channel coefficients. Also, $L_0$ stands for the length of the linear part of the channel (for the channel of [9] $L_0 = 3$). Let us define the vector of output samples $\mathbf{z}_n = [z_n, z_{n-1}, \ldots, z_{n-l}]^T$. Then from (1) we get

$$\mathbf{z}_n = \mathbf{H}_0 \mathbf{x}_{0,n} + \mathbf{H}_1 \mathbf{x}_{1,n} + \cdots + \mathbf{H}_5 \mathbf{x}_{5,n} + \mathbf{w}_n \qquad (2)$$

where $\mathbf{H}_0$ is a convolution (Toeplitz) matrix related to the linear part of the channel, $\mathbf{H}_1$ to $\mathbf{H}_5$ are convolution matrices, which are due to the non-linear channel terms (for the channel of [9] they are diagonal matrices) and the respective vectors are defined appropriately, i.e., $\mathbf{x}_{0,n} = [x_n, x_{n-1}, \ldots, x_{n-L_0-l}]^T$, $\mathbf{x}_{1,n} = [x_n^2 x_{n-2}^*, \ldots, x_{n-l}^2 x_{n-2-l}^*]^T$, etc. It is interesting to note that, based on the Volterra model the output vector $\mathbf{z}_n$ has been expressed in (2) as a sum of several "linear" channel outputs plus noise. Moreover, the nonlinearities have been effectively applied to the input vectors $\mathbf{x}_{i,n}$, for $i = 1, \ldots, 5$.

## IV. MMSE Soft Interference Canceller for Non-Linear Channels

In the scenario under consideration, where the received sequence $z_n$ is corrupted both by nonlinearities and intersymbol interference, we must apply an equalization algorithm able to mitigate both effects. Thus, for equalization of the non-linear channel, we propose the equalizer structure depicted in Fig. 3. The soft output $s_n$ of this equalizer is written as

$$s_n = \mathbf{p}^H \mathbf{z}_n + \mathbf{q}_0^H \mathbf{x}'_{0,n} + \cdots + \mathbf{q}_5^H \mathbf{x}'_{5,n} \qquad (3)$$

where $\mathbf{p} = [p_0, p_1, \ldots, p_l]^T$ is a matched filter, $\mathbf{q}_0$ cancels the ISI due to the linear part of the channel, i.e., $\mathbf{q}_0 = [q_{0,1}, \ldots, q_{0,N_0}]^T$, $\mathbf{x}'_{0,n} = [x_{n-1}, \ldots, x_{n-N_0}]^T$ and filters $\mathbf{q}_1$ to $\mathbf{q}_5$ cancel the ISI due to the corresponding non-linear channel coefficients. In the following, we derive the minimum mean square error filters of this equalizer under the assumption that all cancellation filters contain correct symbols and symbol products. Then, we approximate the probability density function of the soft output $s_n$, which is needed for mapping the soft output $s_n$ to soft outputs about the bits that correspond to $s_n$.

### A. MMSE Filters

By suitably setting the lengths of the cancellation filters according to the input-output model in (1), i.e., $N_0 = l + L_0 = l + 3$, $N_1 = N_2 = \cdots N_5 = 1$ and defining $s_n = \mathbf{v}^H \mathbf{u}_n$ where

$$\mathbf{v} = [\mathbf{p}; \mathbf{q}_0; \cdots; \mathbf{q}_5] \quad \text{and} \quad \mathbf{u}_n = [\mathbf{z}_n; \mathbf{x}'_{0,n}; \cdots; \mathbf{x}'_{5,n}] \ ,$$

we obtain the following solution for the filters of the NL-SIC, that minimizes $E[|x_n - s_n|^2]$

$$\mathbf{v}_o = \mathbf{R}^{-1}\mathbf{r}, \quad \mathbf{R} = E[\mathbf{u}_n \mathbf{u}_n^H], \quad \mathbf{r} = E[x_n \mathbf{u}_n] \ . \qquad (4)$$

It can be seen that the computation of matrix $\mathbf{R}$, involves computation of terms of the following forms,

$$\mathbf{H}_i E[\mathbf{x}_{i,n}\mathbf{x}_{j,n}^H]\mathbf{H}_j^H \quad , \quad \mathbf{H}_i E[\mathbf{x}_{i,n}\mathbf{x'}_{j,n}^H] \ ,$$

$$E[\mathbf{x'}_{i,n}\mathbf{x}_{j,n}^H]\mathbf{H}_j^H \quad \text{and} \quad E[\mathbf{x'}_{i,n}\mathbf{x'}_{j,n}^H] \ .$$

Thus, we turn our attention to the computation of the above expressions. Examining third order symbol products of the form $x_i x_j x_k^*$, we get for PSK symbols with power equal to one

$$x_i x_j x_k^* = \begin{cases} x_j, & \text{if} \quad i = k \\ x_i, & \text{if} \quad j = k \end{cases} \qquad (5)$$

i.e., third order symbol products reduce to first order terms. As a result, all third order symbol products $x_i x_j x_k^*$ of the Volterra model are such that $i \neq k$ and $j \neq k$. Thus, the expectations of symbol products of six terms $x_i x_j x_k^* x_{i'}^* x_{j'}^* x_{k'}$ equal 1 only when $i = i', j = j', k = k'$ $(i = j', j = i', k = k'$ is the same case because it involves exactly the same terms), otherwise they are zero. Also, expectations of four terms symbol products $x_i x_j x_k^* x_{i'}^*$ are always zero. Due to these properties, it follows that,

$$E[\mathbf{x'}_{i,n}\mathbf{x'}_{j,n}^H] = \begin{cases} \mathbf{I}_{N_i}, & \text{if} \quad i = j \\ \mathbf{0}_{N_i \times N_j}, & \text{if} \quad i \neq j \end{cases}$$

Using similar reasoning, it can be shown that $\mathbf{R}$ has the following special form

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{zz} & \mathbf{H}_{0,B} & \mathbf{H}_{1,A} & \cdots & \mathbf{H}_{5,A} \\ \mathbf{H}_{0,B}^H & \mathbf{I}_{N_0} & \mathbf{0}_{N_0 \times 1} & \cdots & \mathbf{0}_{N_0 \times 1} \\ \mathbf{H}_{1,A}^H & \mathbf{0}_{1 \times N_0} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{5,A}^H & \mathbf{0}_{1 \times N_0} & 0 & \cdots & 1 \end{bmatrix} \qquad (6)$$

where

$$\mathbf{R}_{zz} = \mathbf{H}_0 \mathbf{H}_0^H + (|H_{002}^{(3)}|^2 + \cdots + |H_{110}^{(3)}|^2)\mathbf{I}_{l+1} + \sigma_w^2 \mathbf{I}_{l+1} \ ,$$

$\mathbf{H}_{0,B}$ is a matrix consisting of the last $N_0$ columns of $\mathbf{H}_0$, and $\mathbf{H}_{i,A}$ denotes the first column of matrix $\mathbf{H}_i$. Similarly, we have that $\mathbf{r} = [\mathbf{H}_0 \mathbf{d}; \mathbf{0}_{(N_0+5)\times 1}]$, with $\mathbf{d} = [1; \mathbf{0}_{N_0 \times 1}]$.

### B. Input to the Cancellation Filters

It can be noticed from Fig. 3 that the inputs to the cancellation filters are the expected values of linear and non-linear symbol terms. These quantities can be easily computed based on the a-priori information (in the form of log-likelihood ratios) coming from the decoder. For example,

$$E[x_n^2 x_{n-2}^*] = \sum_{i=1}^{2^q} \sum_{j=1}^{2^q} \alpha_i^2 \alpha_j^* \Pr\{x_n = \alpha_i\} \Pr\{x_{n-2} = \alpha_j\} \qquad (7)$$

where, according to the independence assumption, the probabilities $\Pr\{x_n = \alpha_i\}$ are computed as the products of the $q$ bit probabilities that come from the channel decoder and correspond to symbol $\alpha_i$, i.e.,

$$\Pr\{x_n = \alpha_i\} = \prod_{j=0}^{q-1} \Pr\{c_{m+j} = \beta_{i,j}\}$$

In the last expression, $c_{m+j-1}$ denotes the $j$-th bit corresponding to symbol $x_n$ and $\beta_{i,j}$ is the $j$-th bit of symbol $\alpha_i$. It should be noted that the expectation in (7) is with respect to the a-priori probabilities of the symbols provided by the channel decoder.

### C. Output Statistics

In order to transform the output of the NL-SIC into log-likelihood ratios, the mean and variance of $s_n$, given that a particular symbol $\alpha_i$ has been transmitted, must be computed. For these statistics, we get

$$\mu_{i,n} = E[s_n | x_n = \alpha_i] = \alpha_i \mathbf{p}^H \mathbf{H}_0 \mathbf{d} \ , \qquad (8)$$

and for the variance, omitting the contribution of third order terms for simplicity,

$$\sigma_{i,n}^2 = \sigma_w^2 \mathbf{p}^H \mathbf{p} + \mathbf{q}_0^H \mathbf{V}_n \mathbf{q}_0 \ . \qquad (9)$$

$\mathbf{V}_n$ is a diagonal matrix containing the variances of symbols $x_{n-1}$ to $x_{n-N_0}$ that are computed using a-priori probabilities coming from the channel decoder, in a manner similar to equation (7). Finally, under the assumption that the output of the canceller is normally distributed, the above computed statistics are used to transform the soft output $s_n$ into log likelihood ratios, via

$$L_e^{(E)}(c_m) = \ln\left(\frac{\sum_{\beta_{i,j}=1} Pr\{x_n = a_i\} p(s_n | x_n = a_i)}{\sum_{\beta_{i,j}=0} Pr\{x_n = a_i\} p(s_n | x_n = a_i)}\right) \ ,$$

where $\beta_{i,j}$ $(i = 1, \ldots, 2^q,\ j = 0, \ldots, q-1)$ denotes the $j$-th bit for symbol $\alpha_i$ and $p(s_n | x_n = a_i)$ is a Gaussian p.d.f with mean $\mu_{i,n}$ and variance $\sigma_{i,n}^2$.

## V. SIMULATION RESULTS

In order to investigate the effectiveness of the proposed scheme in mitigating the non-linear ISI, computer simulations were conducted. Information bits were generated in packets of S=6144 bits. The information bits were protected using a rate $1/2$ recursive systematic convolutional (RSC) code with octal generator matrix $G = [1 \quad 5/7]$. The resulting bits were

| Channel Coefficient | Value |
|---|---|
| $H_0^{(1)}$ | 0.8529 + 0.4502i |
| $H_1^{(1)}$ | 0.0881 - 0.0014i |
| $H_2^{(1)}$ | -0.0336 - 0.0196i |
| $H_3^{(1)}$ | 0.0503 + 0.0433i |
| $H_{002}^{(3)}$ | 0.1091 - 0.0615i |
| $H_{330}^{(3)}$ | 0.0503 - 0.0503i |
| $H_{001}^{(3)}$ | 0.0979 - 0.0979i |
| $H_{003}^{(3)}$ | -0.1119 - 0.0252i |
| $H_{110}^{(3)}$ | -0.0280 - 0.0475i |

TABLE I

THE NON-LINEAR CHANNEL USED IN THE SIMULATIONS

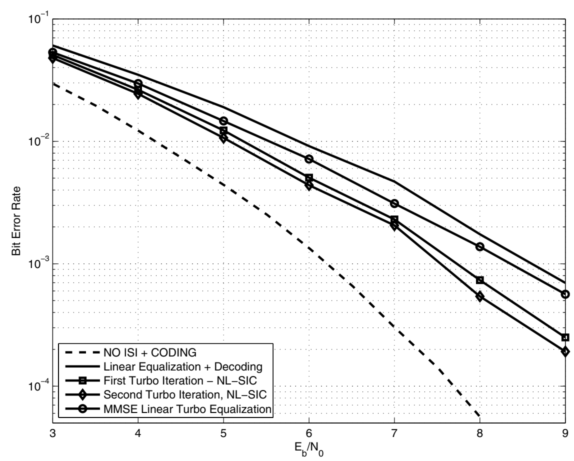


Fig. 4. BER vs SNR for various TE schemes

interleaved using an $\mathcal{S}$-random interleaver ($\mathcal{S} = 23$). The interleaved bits were mapped to 8-PSK symbols using Gray code mapping. The resulting 4096 symbols per packet were transmitted over a non-linear channel that was based upon the channel derived in [9]. In particular, in order to increase the ISI introduced by the channel of [9], the linear ISI terms were multiplied by 2 and the non-linear ISI terms by 4. The resulting channel coefficients, normalized to deliver unit symbol energy at the receiver, appear in Table I. At the receiver, we employed turbo equalization using the maximum a-posteriori (MAP) channel decoder and various soft-input soft-output equalizers. At the first iteration of all the examined methods, a linear equalizer has been employed.

Fig. 4 shows that due to the channel nonlinearities, the MMSE linear turbo equalizer of [4] performs poorly and can not go beyond a certain lower limit even after a high number of turbo iterations. On the other hand, the proposed soft interference canceller exhibits superior performance even after one iteration only. This is due to the fact that nonlinear terms, ignored by the linear equalizer, are taken into account by the proposed soft interference canceller. As a benchmark, the performance of the ISI-free AWGN channel with coding is also provided.

## VI. CONCLUSION

In this work, a soft interference canceller for non-linear Volterra channels has been proposed. The filters of the proposed equalizer were optimized using the minimum mean square error criterion, and the assumption that the cancellation filters of the canceller contain correct estimates. Simulation results have shown that, under a non-linear channel setup, by incorporating the NL-SIC into a turbo equalization scheme, noticeable performance gains over conventional linear TE schemes can be obtained.

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