ENERGY-BASED MODEL-INDEPENDENT SOURCE LOCALIZATION IN WIRELESS SENSOR NETWORKS

D. Ampeliotis and K. Berberidis

Computer Engineering and Informatics Department and RACTI/RU8
University of Patras, 26500, Rio-Patras, Greece
phone: + 30 2610 996973, fax: + 30 2610 996971,
emails: {ampeliot, berberid}@ceid.upatras.gr

ABSTRACT

Localization of an isotropic source using energy measurements from distributed sensors is considered. Usually, such localization techniques require that the distances between the sensor nodes and the source of interest have been previously estimated. This, in turn, requires that sufficient information about the energy decay model as well as the transmit power of the source is available. In this work, making the assumption that the locations of nodes near the source can be well described by a uniform distribution, we derive distance estimates that are independent of both the energy decay model and the transmit power of the source. Numerical results show that these estimates lead to improved localization accuracy as compared to other model-independent approaches.

1. INTRODUCTION

Wireless sensor networks have recently received great attention because they hold the potential to change many aspects of our economy and life. Among many applications, ranging from environmental monitoring to manufacturing, source localization and tracking has been widely viewed as a canonical problem of wireless sensor networks. Furthermore, it constitutes an easily perceived application that can be used as a vehicle to study more involved information processing and organization problems [13]. On the other hand, the design, implementation and operation of a sensor network requires the synergy of many disciplines, including signal processing, networking and distributed algorithms. Moreover, sensor networks must operate using minimum resources: typical sensor nodes are battery powered and have limited processing ability. These constraints impose new challenges in algorithm development, and imply that power efficient, distributed and cooperative techniques should be employed.

Most of the source localization methods that have appeared in the literature can be classified into two broad categories. The algorithms of the first category utilize Time Delay Of Arrival (TDOA) measurements, whereas the algorithms of the second category use Direction Of Arrival (DOA) measurements. DOA estimates are particularly useful for locating sources emitting narrowband signals [5], while TDOA measurements offer the increased capability of localizing sources emitting broadband signals [7]. However, the methods of both categories impose two major requirements that render them unappropriate to be used in wireless sensor networks, i.e.: (a) The analog signals at the outputs of the spatially distributed sensors should be sampled in a synchronized fashion, and (b) the sampling rate used should be high enough so as to capture the features of interest. These requirements, in turn, imply that accurate distributed synchronization methods should be implemented so as to keep the remote sensors synchronized and that high frequency electronics as well as increased bandwidth are needed to communicate the acquired measurements.

Recently [6], a new approach to source localization was proposed, that utilizes Received Signal Strength (RSS) measurements. In particular, the spatially distributed sensors measure the power of the signal due to the source that arrives at their location. In the sequel, using an energy-decay model, each sensor is able to extract some information about its distance to the source of interest. Finally, the required location of the source is derived by proper fusion of the information extracted at a number of *active* sensor nodes. Note that, a sensor node is characterized as active if its measurement is greater than a predetermined threshold. In [6], in order to avoid the ambiguities that arise due to the unknown transmit power of the source, it was proposed to compute ratios of measurements taken at pairs of active sensors. In [10], maximum likelihood multiple-source localization based on RSS measurements was considered. In [12], the problem of source localization was formulated as a coverage problem and estimates of the necessary sensor density which can guarantee a localization error bound were derived. In [8], a distributed "incremental subgradient" algorithm was prosposed to yield iteratively the source location estimate. More recently, a distributed localization algorithm enjoying good convergence properties was proposed in [2]. In [4], a non-linear cost function for localization was proposed and it was proved that its gradient descent minimization is globally converging. However, all the aforementioned approaches require knowledge of the energy decay model and/or the transmit power of the source of interest.

In [9], the case of unavailable information about the energy decay model and the transmit power of the source (i.e. *model-independent* case) was considered. The location of the source was derived by properly averaging the locations of active sensor nodes. Note at this point that the work in [9], similarly to our approach, also uses the assumption of uniform deployment of sensors over the field of interest. Another model-independent localization method, that can also be viewed as a special case of the aforementioned estimator, is to detect the sensor node with the strongest energy measurement and set the location estimate equal to its location [6], assuming that this node is the closest one to the source, the so-called Closest Point of Approach (CPA).

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In this work, we propose an alternative model-independent estimator. In particular, we derive the Probability Density Functions (PDFs) of the random variables describing the distances between the source and the k-th closest sensor node to the source. Thus, if an active sensor node knows its rank k, it can obtain the PDF of its distance from the source. A distributed sorting algorithm, such as the ones proposed in [3] and [11], can be used so that all active nodes can obtain their rank, by assuming that active nodes with higher RSS measurement are closer to the energy source. In the sequel, once all active nodes know their rank, the required distance estimates are obtained as the expected values of the respective PDFs. Finally, the distance estimates thus obtained can be used by the so-called "Projection Onto Convex Sets" (POCS) approach of [2], to yield an estimate of the location of the source of interest. Simulation results have shown that, in many cases, the proposed model-independent localization method offers improved accuracy as compared to other model-independent approaches that rely upon exactly the same assumptions.

2. PROBLEM FORMULATION

Let us consider that N sensor nodes have been deployed uniformly at random over a territory of interest. Let us also consider that an energy source is present in the same area. Denote the 2×1 location vector of the n-th sensor node as $\mathbf{r}_n$ and the unknown location vector of the source as $\mathbf{r}$. Each sensor takes measurements according to the model

$$y_n = \alpha\, g(||\mathbf{r}_n - \mathbf{r}||) + w_n \qquad n = 1,2,\ldots,N \tag{1}$$

where $\alpha > 0$ denotes the strength of the source, function $g : \mathscr{R}^+ \to \mathscr{R}^+$ is monotone decreasing and w_n denote zero mean statistically independent noise terms. Thus, the localization problem in our context is defined as: Assuming a function $g(\cdot)$ as above and given the RSS measurements y_n and the location vectors $\mathbf{r}_n$ of the sensor nodes, provide an estimate $\hat{\mathbf{r}}$ of the location vector of the source.

Recently, it has been made clear that the solution to the localization problem can be computed very efficiently provided that the distances between the sensor nodes and the source of interest have been previously estimated. In particular, [2] and [4] provide globally converging algorithms for obtaining an estimate of the location of the source in such a case.

However, it should be stressed that [2] and [4] presume that the required distance quantities, i.e. $||\mathbf{r}_n - \mathbf{r}||$, are somehow available. Commonly, the estimation of these distances is based on proper assumptions with respect to the form of function $g(\cdot)$. In many applications, function $g(\cdot)$ takes the usual form

$$g(x) = \frac{1}{x^\beta}\ , \tag{2}$$

where β is denoted as the energy decay exponent with typical values in the range $\beta \in [2,4]$. From (1) and (2), it can be seen that if the parameters α and β are known, then each node can estimate its distance from the source using

$$||\mathbf{r}_n - \mathbf{r}|| \approx (\alpha / y_n)^{1/\beta}\ . \tag{3}$$

However, in practice, the aforementioned parameters may not be known. Furthermore, it is also possible that function $g(\cdot)$ does not take the form in (2). In such a case, one could resort to the averaging estimator proposed in [9], or its special case, termed the CPA estimator, mentioned in [6]. In the following, we provide an estimator of the distances for the model-independent case, so that the approaches in [2] and [4] can also be applied.

3. MODEL-INDEPENDENT LOCALIZATION

We will start the derivation of the new estimator by first making a useful remark: Consider any fixed deployment of N sensors over a territory of interest, where a fixed source also exists. Consider also two different energy decay functions $g_1(\cdot)$ and $g_2(\cdot)$ and let us ignore noise for the time being. Since we want to develop a model-independent estimator, it follows that our estimator would give the same result in both cases. Considering now the RSS measurements that result from the two different models, we note that the only common information between the two sets of measurements is simply their ordering and not their actual values, since the only knowledge we have about the energy decay functions is that they are monotone decreasing. Thus, we deduce that the only useful information in the model-independent case is contained in the ordering of the measurements and not their actual values. In other words, in the model-independent case, the ordering of the RSS measurements is a sufficient statistic for the estimation of the location of the source.

In the following paragraphs, we first derive the PDFs of the distance of the k-th closest sensor to the source, making the assumption that the locations of nodes near the source can be well described by a uniform distribution. In the sequel, distance estimates are derived as the expected values of these PDFs. Then, we give a brief description of the POCS method [2], which is a constituent part of the proposed localization algorithm. Finally, the proposed model-independent localization method is summarized.

3.1 Derivation of the PDFs of the distances

Let us define an area E in which N sensors have been uniformly deployed. The spatial density of this network is defined as $d = N/E$. Let us also consider that a single source exists in E, and define a circular area R of radius ρ around that source. Define also the random variable $\mathscr{P}_k$ describing the distance of the k-th closest sensor to the source. Then, the probability that the distance of the k-th sensor away from the source is smaller than ρ, is given by:

$$\begin{aligned} Pr\{\mathscr{P}_k \le \rho\} &= Pr\{\#\text{ sensors in } R \ge k\} \\ &= 1 - Pr\{\#\text{ sensors in } R < k\} \\ &= 1 - \sum_{i=0}^{k-1} Pr\{\#\text{ sensors in } R = i\} \end{aligned} \tag{4}$$

where we have used the symbol # as an abbreviation for the expression "number of". The required probabilities $q_i = Pr\{\#\text{ sensors in } R = i\}$ can be computed by considering that each one of the N nodes, independently, either lies in R or outside R with probabilities equal to R/E and $(E-R)/E$ respectively, and that there are $\binom{N}{i}$ different realizations in which i sensors lie in R, thus

$$q_i = \left[\left(\frac{\pi\rho^2}{E}\right)^i \left(1 - \frac{\pi\rho^2}{E}\right)^{N-i} \binom{N}{i} \right]$$

$$= \left[\left(\frac{\pi d\rho^2}{N}\right)^i \left(1-\frac{\pi d\rho^2}{N}\right)^{N-i} \binom{N}{i}\right] . \quad (5)$$

Asymptotically, as N and $E = N/d$ tend to infinity (i.e. the network covers all the plane, but with constant density), after some algebra we have that:

$$\begin{aligned}\lim_{N\to+\infty}[q_i] &= \frac{(\pi d\rho^2)^i}{i!}\cdot \\ &\cdot \lim_{N\to+\infty}\left[\frac{N(N-1)\cdots(N-i+1)}{(N-\pi d\rho^2)^i}\right]\cdot \\ &\cdot \lim_{N\to+\infty}\left[\left(1-\frac{\pi d\rho^2}{N}\right)^N\right] \\ &= \frac{(\pi d\rho^2)^i}{i!}e^{-\pi d\rho^2} . \end{aligned} \quad (6)$$

Thus, the Cumulative Density Function (CDF) of the random variable $\mathscr{P}_k$ describing the distance of the k-th closest sensor to the source will be given by:

$$F_{\mathscr{P}_k}(\rho) = 1-\sum_{i=0}^{k-1}\frac{(\pi d\rho^2)^i}{i!}e^{-\pi d\rho^2} . \quad (7)$$

The PDF of $\mathscr{P}_k$ will thus be given as the derivative of the above CDF as:

$$\begin{aligned} f_{\mathscr{P}_k}(\rho) &= F'_{\mathscr{P}_k}(\rho) \\ &= 0-\sum_{i=0}^{k-1}\frac{1}{i!}\{(\pi d)^i 2i\rho^{2i-1}e^{-\pi d\rho^2} \\ &\quad +(\pi d\rho^2)^i e^{-\pi d\rho^2}(-2\pi d\rho)\} \\ &= \sum_{i=0}^{k-1}\frac{2\pi^{i+1}d^{i+1}\rho^{2i+1}}{i!}e^{-\pi d\rho^2} \\ &\quad -\frac{2i\pi^i d^i\rho^{2i-1}}{i!}e^{-\pi d\rho^2} . \end{aligned} \quad (8)$$

Defining now the sequence

$$a_i = \frac{2\pi^{i+1}d^{i+1}\rho^{2i+1}}{i!}e^{-\pi d\rho^2} \quad (9)$$

we have for $k = 1$ that

$$f_{\mathscr{P}_1}(\rho) = a_0 = 2\pi d\rho e^{-\pi d\rho^2} \quad (10)$$

and for $k > 1$ that

$$f_{\mathscr{P}_k}(\rho) = a_0 + \sum_{i=1}^{k-1} a_i - a_{i-1} = a_{k-1} . \quad (11)$$

Thus, we finally have for all k that the corresponding PDF is given by

$$f_{\mathscr{P}_k}(\rho) = a_{k-1} = \frac{2\pi^k d^k}{(k-1)!}\rho^{2k-1}e^{-\pi d\rho^2} \quad (12)$$

which concludes our derivation.

3.2 Distance Estimation

Using the PDFs derived in the previous paragraph, the required distances can be obtained by computing the respective expected values. Using the form of (12) for $f_{\mathscr{P}_k}(\rho)$ and Euler's integral, (see Section 6.1.1 of [1]), it can be shown that:

$$E[\mathscr{P}_k] = \int_0^{+\infty}\rho f_{\mathscr{P}_k}(\rho)d\rho = \frac{1}{\sqrt{\pi d}}\frac{\Gamma(k+1/2)}{\Gamma(k)} , \quad (13)$$

where the ratio due to the Gamma function could be precomputed and stored in a lookup table. Alternatively, the square root of the expected value of the square of the random variable $\mathscr{P}_k$ can be used, that yields a simpler formula

$$\sqrt{E[\mathscr{P}_k^2]} = \left(\int_0^{+\infty}\rho^2 f_{\mathscr{P}_k}(\rho)d\rho\right)^{1/2} = \sqrt{\frac{k}{\pi d}} . \quad (14)$$

Simulation results presented in Section 4 demonstrate negligible performance degradation when equation (14) is used instead of (13).

3.3 Localization via POCS

In [2], source localization using RSS measurements was formulated as a convex feasibility problem. In particular, provided that all active sensors have an estimate of their distance to the source, a disk D_n (convex set) was defined for each one of them, with its center at $\mathbf{r}_n$ and radius equal to the estimated distance. Thus, it was proposed to solve the localization problem by letting the estimator be a point in the intersection of the sets D_n:

$$\hat{\mathbf{r}} \in D = \bigcap_{n\in\mathscr{A}} D_n , \quad (15)$$

where $\mathscr{A}$ denotes the set of active nodes.

Having expressed the localization problem as in (15), it was then proposed to apply the Projection Onto Convex Sets approach, to yield a solution. This procedure is known to converge to a point in D when this set is non-empty, or, when D is empty, it converges to a limit cycle in the vicinity of the point that minimizes the sum of distances to the sets D_n. POCS updates the estimate θ_t of the location of the source at time t (initialized to an arbitrary vector $\theta_0 \in \mathscr{R}^2$) to the estimate θ_{t+1}, using

$$\theta_{t+1} = \theta_t + \lambda_t(B_{D_n}(\theta_t) - \theta_t) \quad (16)$$

where $B_{D_n}(\cdot)$ is a function that returns the projection of its vector argument onto the set D_n, i.e. the nearest point (using Euclidean distance) to θ_t that belongs to D_n. Also, λ_t is a sequence of properly chosen relaxation parameters [2]. In the case of source localization, a great advantage of POCS comes from the fact that function $B_{D_n}(\cdot)$ can be computed in closed form.

3.4 Summary of model-independent localization

In this subsection, we summarize the proposed localization algorithm. The algorithm consists of the following steps:

1. The set of active nodes $\mathscr{A} = \{n_1, n_2, \ldots, n_L\}$ is defined, consisting of all nodes with measurement $y_{n_i} > T$, where T a properly selected threshold.

2. Nodes in $\mathscr{A}$ run a distributed sorting algorithm, as the ones proposed in [3] and [11]. The scope of this step is to compute the ranks k_{n_i} of active sensors, assuming that nodes with higher energy measurement are closer to the energy source.
3. Nodes in $\mathscr{A}$ compute estimates $\hat{\rho}_{n_i}$ of their distance to the source using k_{n_i} and formula (13) or (14).
4. The active nodes are organized in a circle. A node, chosen as the first one, initializes the location estimate θ_0 to an arbitrary 2×1 vector.
5. Each node n_j that receives an estimate θ_t from the previous node, checks if $||\mathbf{r}_{n_j} - \theta_t|| < \hat{\rho}_{n_j}$, and if not, it updates the estimate using [2]:
   5.1) $\phi = atan(\theta_t(2) - \mathbf{r}_{n_j}(2), \theta_t(1) - \mathbf{r}_{n_j}(1))$
   5.2) $\mathbf{b}_{t+1} = \mathbf{r}_{n_j} + \hat{\rho}_{n_j}[\cos(\phi) \quad \sin(\phi)]^T$
   5.3) $\theta_{t+1} = \theta_t + \lambda_t\,[\mathbf{b}_{t+1} - \theta_t]$
   where $atan(\cdot,\cdot)$ is the four quadrant inverse tangent function, and for a vector $\mathbf{x} \in \mathscr{R}^2$, $\mathbf{x}(1)$ and $\mathbf{x}(2)$ denote its first and second coordinates, respectively.
6. Node n_j forwards θ_{t+1} to the next node, until a fixed number of circles has been completed.

4. NUMERICAL RESULTS

In order to assess the performance of the proposed algorithm we performed some numerical simulations. In particular, N nodes were uniformly deployed over a $100m \times 100m$ field, where N was increased from 300 to 3100 in 200 increments. A signal source with $\alpha = 100$ was located at $\mathbf{r} = [50\ 50]^T$, i.e. at the center of the deployment field. The RSS measurements at the sensor nodes were corrupted by zero mean Additive White Gaussian Noise (AWGN) with variance $\sigma^2 = 1$. A threshold $T = 5$ was used to detect active nodes, i.e. only sensors whose SNR (y_n/σ^2) is greater than 7dB take part in the estimation procedure. We examined four different energy decay functions, in particular the one of equation (2) with $\beta = 2$, the one of (2) with $\beta = 3$, the one having

$$g(x) = e^{-x} \tag{17}$$

and the one having

$$g(x) = e^{-x/2}\ . \tag{18}$$

In all cases, 5 different localization algorithms were examined. In particular, the proposed POCS-based algorithms using (13) and (14) were compared against the Averaging estimator proposed in [9] and the CPA estimator mentioned in [6]. As a benchmark, the POCS estimator that has perfect knowledge of the energy decay function and the power of the source was also included. For all POCS-based algorithms, we used the constant relaxation sequence $\lambda_t = 1$, while the final estimate was given as the average of the estimates during the last cycle, as was also done in [2]. We performed a total of 10 cycles. Also note that for the averaging estimator of [9] we did not use any optimized kernel.

Figures 1.(a) to 1.(d) show the Root Mean Square (RMS) error obtained for the aforementioned energy decay models, respectively, as a function of the average number of nodes that entered the estimation phase. Note that, since the same threshold T is used for all the examined energy decay models, the respective average numbers of active nodes differ significantly. For the last two energy decay models, in the case where we have full knowledge of their parameters (i.e. dashed lines), distance estimation was done using the relations

$$||\mathbf{r}_n - \mathbf{r}|| \approx -\ln(y_n/\alpha) \tag{19}$$

and

$$||\mathbf{r}_n - \mathbf{r}|| \approx -2\ln(y_n/\alpha) \tag{20}$$

respectively.

Three are the main observations that can be made from Figure 1:

1. As already mentioned, negligible performance degradation is obtained by using equation (14) in place of equation (13). Thus, it is concluded that it is not necessary that sensor nodes should be able to compute values of the Gamma function or keep a lookup table.
2. It is seen that for all of the examined energy decay models, the proposed algorithm yields smaller localization error than the other model-independent localization methods, if more than five sensors are on average active.
3. The proposed localization algorithm approaches the performance of the POCS algorithm with perfect knowledge of the energy decay model and the power of the source, as the average number of active nodes is increased. This behavior is even more noticeable when the received energy decays slower with distance (i.e. cases (a) and (d)).

5. CONCLUSIONS

Localization of an isotropic energy source using measurements from distributed sensors was considered. The proposed algorithm does not assume any knowledge of the energy decay function or the power of the source. Instead, making the assumption that the locations of nodes near the source can be well described by a uniform distribution, distances between the sensors and the source of interest that are invariant to both the energy decay model and the transmit power of the source were derived. The method of Projections Onto Convex Sets was employed to yield an estimate of the source location. Numerical results have shown that these estimates lead to improved localization accuracy as compared to other model-independent approaches and furthermore, the accuracy of the proposed method is very close to the accuracy of other techniques that have perfect knowledge of the energy decay model and the power of the source.

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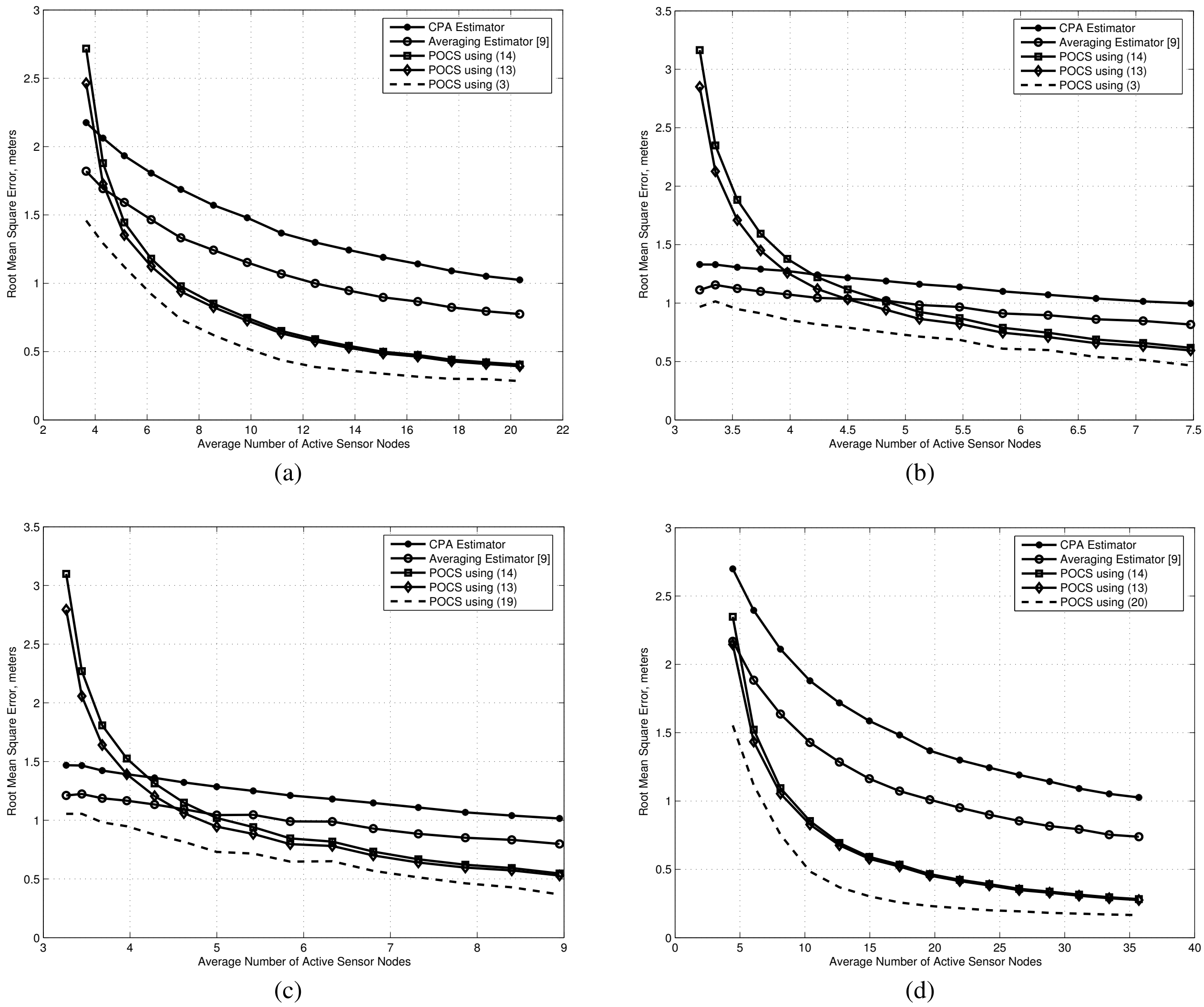

Figure 1: RMS error as a function of the average number of active nodes for the examined localization algorithms and various energy decay functions: (a) $g(x) = 1/x^2$ (b) $g(x) = 1/x^3$ (c) $g(x) = e^{-x}$ (d) $g(x) = e^{-x/2}$

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