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Low complexity multiple acoustic source localization in sensor networks based on energy measurements

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ABSTRACT

This work addresses the problem of estimating the locations of multiple acoustic sources by a network of distributed energy measuring sensors. The maximum likelihood (ML) solution to this problem is related to the optimization of a non-convex function of, usually, many variables. Thus, search-based methods of high complexity are required in order to yield an accurate solution. Considerable reduction of the complexity can be achieved by means of an alternating projection (AP) algorithm that decomposes the original problem into a number of simpler, yet also non-convex, optimization steps. The particular form of the derived cost functions of each such optimization step indicates that, in some cases, an approximate form of these cost functions can be used. These approximate cost functions can be evaluated using considerably lower computational complexity. Thus, a low-complexity version of the AP algorithm is proposed. Extensive simulation results demonstrate that the proposed algorithm offers a performance close to that of the exact AP implementation, and in some cases, similar performance to that of the ML estimator.

1. Introduction

Technology advances in microelectronics and wireless communications have enabled the development of small scale devices that integrate sensing, processing and short range radio capabilities. The deployment of a large number of such devices, referred as sensor nodes, over a territory of interest, defines the so-called wireless sensor network (WSN) [1,2]. WSNs have attracted considerable attention in recent years and have motivated many new challenges, most of which require the synergy of many disciplines, including signal processing, networking and distributed algorithms. Among many other applications, source localization and tracking has been widely viewed as a canonical problem of wireless sensor networks. Furthermore, it constitutes an easily perceived problem that can be used as a vehicle to study more involved information processing and organization problems [1].

In this work, we deal with the problem of source location estimation using passive and stationary acoustic sensors. Such source localization methods have a wide range of possible applications. Indoor applications include the localization and tracking of human speakers for the purposes of video conferencing. In an outdoor environment, a WSN deployed in an open field can utilize the sound emitted from a moving vehicle in order to track its location, for example in surveillance applications. In sonar signal processing, the focus can be the localization of underwater acoustic sources (i.e. sea mammals, divers) using an array of hydrophones.

Most of the source localization methods that have appeared in the literature can be classified into two broad categories, according to the physical variable they utilize. The algorithms of the first category utilize time-delay-of-arrival (TDOA) measurements, whereas the algorithms of

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the second category use direction-of-arrival (DOA) measurements. DOA estimates are particularly useful for locating sources emitting narrowband signals [3], while TDOA measurements offer the increased capability of localizing sources emitting broadband signals [4,5]. However, the methods of both categories impose two major requirements that render them unappropriate to be used in wireless sensor networks: (a) the analog signals at the outputs of the spatially distributed sensors should be sampled in a synchronized fashion and (b) the sampling rate used should be high enough so as to capture the features of interest. These requirements, in turn, imply that accurate distributed synchronization methods should be implemented so as to keep the remote sensor nodes synchronized and that high frequency electronics as well as increased bandwidth are needed to transmit the acquired measurements.

Due to the aforementioned limitations, source localization methods that rely upon received signal strength (RSS) measurements—originally explored for locating electromagnetic sources [6,7]—have recently received revived attention [8–25]. These approaches may be categorized as follows:

(a) Single source localization algorithms, which can be further divided into:
  (a1) Algorithms that rely upon a known energy decay model.
  (a2) Model-independent localization algorithms that assume a general, unknown, monotone decreasing energy decay function.
(b) Multiple source localization algorithms.

Let us briefly review the existing literature under the above categorization.

Category (a1): In [8], an energy decay model for acoustic sources in free air was proposed and verified experimentally. Also, in order to avoid ambiguities due to the unknown power of the source, a localization algorithm that computes ratios of measurements was reported. In [9], the authors considered an incremental subgradient optimization algorithm to yield the source location in a decentralized fashion. In [10] the authors considered various least squares based criteria for localization. The authors of [11] proposed a linear least squares localization algorithm, as an extension of the one proposed in [12], and considered its distributed implementation. In [14], the problem of single source localization with a known energy decay model was formulated as a convex feasibility problem. Thus, the method of projections onto convex sets (POCS) was applied to yield a fast converging algorithm that was also shown to be amenable to distributed implementation. In [15], the algorithms proposed in [8] were extended by means of proper weighting schemes. The authors of [16] identified conditions under which gradient descend minimization of a least squares criterion is globally converging. More recently, the authors of [17] proposed the so-called normalized incremental subgradient algorithm. In [18], the authors considered various weighted least squares based algorithms, proved the equivalence of some algorithms previously reported, and proposed a novel least squares algorithm. In [19], linear least squares based algorithms able to estimate the source location and its power were reported.

Category (a2): In contrast to the abundance of the works presented in the previous paragraph, the literature on localization algorithms that do not assume a specific form for the energy decay model is quite more limited. In particular, in [8], the closest point of approach (CPA) estimator was presented. This simple method sets the location of the sensor node with the highest energy measurement as the estimate for the location of the source. Thus, no information about the energy decay model is required. The expected performance of this estimator was studied in [20]. In [21], it was proposed to estimate the location of the source as the average of the locations of nodes whose measurements are above a predefined threshold. More recently [22], the probability density functions of the distances between the source and the closest, second closest, etc., sensor nodes were derived. Also, it was experimentally shown that the utilization of the expected values of the distances thus obtained into a POCS based algorithm leads to a performance very close to that of algorithms that have accurate knowledge of the energy decay model.

Category (b): Similarly to the previous category, the literature on multiple source localization from RSS measurements is quite limited. In [23], the maximum likelihood estimator was studied. As already mentioned, the optimization problem related to ML estimation is non-convex, and thus, search-based methods are required in order to yield an accurate solution. In [24], a particle filter based algorithm was proposed. The so-called importance function of the proposed particle filter was derived using results from [23]. Also, in [25], two methods for the distributed implementation of the particle filter were proposed. In [26], the authors consider an alternating projection algorithm, where each optimization step is performed using multi-resolution search. Thus, complexity savings due to both the alternating projection approach and multi-resolution search can be gained.

In this work, we elaborate on the alternating projection method in order to further decrease its computational complexity. In particular, in contrast to the work in [26], we employ the projection matrix update formula for deriving the functions that need to be maximized at each step of the AP scheme. Also, we show that each evaluation of the cost function that is used in the AP approach requires $O(N_a^2)$ operations, where N_a denotes the number of sensors that participate in the estimation. Thus, in the case where N_a is large, the evaluation of the cost function at a number of search points would require much computational effort. However, a careful inspection of the form of the cost function reveals that, in some cases, an approximate cost function that requires $O(N_a)$ operations for each evaluation can be utilized. Motivated by this fact, we propose a modified version of the AP algorithm that, using a suitable criterion, can select either the exact cost function or its approximate form, before proceeding to each optimization step. Furthermore, a computationally efficient initialization method is also proposed.

The remaining of this work is organized as follows: In Section 2 the formulation of the problem at hand is given. In Section 3, we briefly review the maximum likelihood and the alternating projection estimators. Also, in Section 3.3, a detailed complexity analysis of the AP scheme is given. In Section 4, the suggested approximate cost functions are presented and a criterion for choosing between the approximate and the exact cost functions is introduced. Also, in Section 4.3, we give a computationally efficient method for initializing the location estimates, by proposing a suitable multi-resolution optimization of the exact cost functions. This initialization method is used by both the exact and the approximate AP algorithms. In Section 4.4, we give a short note regarding the distributed implementation of the examined algorithms. Finally, in Section 5 we explore the performance of the algorithms via numerical simulations, and in Section 6 we draw our conclusions.

2. Problem formulation

Consider that N sensor nodes have been deployed over a p-dimensional ($p \in \{2,3\}$) region of interest and define as $\mathbf{r}_n \in \mathcal{R}^{p\times 1}, n=1,2,\ldots,N$ the vectors that represent their respective locations. Each node is equipped with an acoustic energy measuring sensor. Consider also that within the same region there exist K isotropic acoustic sources and define their location vectors as $\mathbf{x}_k \in \mathcal{R}^{p\times 1}$, $k=1,2,\ldots,K$. Adopting the energy decay model of [8], the RSS measurement at the n-th sensor can be modeled by the relation

$$y_n = g_n \sum_{k=1}^{K} \frac{A_k}{\|\mathbf{r}_n - \mathbf{x}_k\|^{\beta}} + w_n, \quad n=1,2,\ldots,N, \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm, A_k is the strength of the k-th source as measured at 1 m distance, β is the attenuation exponent and w_n denotes zero mean additive white Gaussian noise (AWGN) with variance σ^2. Also, g_n denotes the so-called gain of each sensor. We have assumed that the variance of the noise is equal at all sensors for simplicity reasons, however, the results presented here can easily be generalized in the case where each sensor measurement is corrupted by noise of different (known) variance. The attenuation exponent for acoustic signals traveling in free air can be approximated by $\beta=2$, whereas larger values of β are used to model the energy decay of electromagnetic signals. In this work we will treat β as a known constant, which is suitable for the specific localization application (e.g. for localizing open field acoustic sources). Also, we assume well calibrated sensor nodes so that $g_n=1$. Thus, the localization problem consists in estimating the unknown vectors $\mathbf{x}_k$, $k=1,2,\ldots,K$ and the parameters A_k, $k=1,2,\ldots,K$, given a single frame of measurements $\{y_1,y_2,\ldots,y_N\}$ and the known sensor location vectors $\mathbf{r}_n$, $n=1,2,\ldots,N$.

Although the energy decay model in (1) appears quite simplistic, it is the one commonly used in literature. The main limitations it poses are, its inability to take into account sound reverberations due to possible obstacles, the fact that sound sources may not be isotropic, and that it assumes point sources. For a more detailed discussion on acoustic energy decay models, the interested reader is referred to [8,18].

Inspection of the model of Eq. (1) reveals that not all sensor nodes experience the same signal to noise ratio. Signal strength measurements at sensors that lie near a source are much more "informative" than measurements acquired at sensors away from all sources. Thus, it has been proposed [8,29] that only sensor nodes of relatively high signal-to-noise ratio should take part in the estimation procedure. These nodes define the set of *active nodes* which we denote as $\mathcal{A}$. Usually, each node independently decides if it belongs in $\mathcal{A}$ by testing if its measurement is greater than a predefined threshold T, that is

$$\mathcal{A} = \{n : y_n > T\}. \tag{2}$$

However, in general, the decision about the state (active or inactive) of a sensor node may also depend on information available at other nodes. In such a case, decision fusion techniques [30] may be applied.

3. Estimation techniques for multiple source localization

3.1. The ML estimator

In [23], the maximum likelihood estimator for multiple source localization using energy measurements was formulated. In particular, if we write the $N_a = |\mathcal{A}|$ measurements of active nodes in (1) in vector form, maximum likelihood estimation is equivalent to minimizing the cost function

$$l(\boldsymbol{\theta}) = \|\mathbf{y} - \mathbf{H}\mathbf{a}\|^2, \tag{3}$$

where $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_{N_a}]^T$ is the vector with the measurements. Matrix $\mathbf{H}$ and vector $\mathbf{a}$ depend on the unknown parameters vector $\boldsymbol{\theta} = [\mathbf{x}_1^T \ldots \mathbf{x}_K^T \, A_1 \ldots A_K]^T$, via the relations

$$\mathbf{a} = [A_1 \ A_2 \ \cdots \ A_K]^T \quad \text{and} \quad \mathbf{H} = [\mathbf{h}(\mathbf{x}_1) \ \mathbf{h}(\mathbf{x}_2) \ \cdots \ \mathbf{h}(\mathbf{x}_K)] \tag{4}$$

with

$$\mathbf{h}(\mathbf{x}) = \left[\frac{1}{\|\mathbf{r}_1 - \mathbf{x}\|^2} \frac{1}{\|\mathbf{r}_2 - \mathbf{x}\|^2} \cdots \frac{1}{\|\mathbf{r}_{N_a} - \mathbf{x}\|^2}\right]^T. \tag{5}$$

Noting that vector $\mathbf{a}$ participates in (3) linearly, we can replace it by its least squares estimate for given $\mathbf{H}$, which is given by

$$\mathbf{a}_o = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{y} \tag{6}$$

so as to get the modified cost function

$$l'(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_K) = \|\mathbf{y} - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{y}\|^2 \tag{7}$$

which is equivalent to (3) in terms of their global minimum. Minimization of (7) can be transformed into an equivalent maximization problem:

$$\mathbf{H}_o = \underset{\mathbf{H}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{y}\|^2 = \underset{\mathbf{H}}{\operatorname{argmax}} \mathbf{y}^T\mathbf{P}_{\mathbf{H}}\mathbf{y}^T, \tag{8}$$

where $\mathbf{P}_{\mathbf{H}} = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$ is a projection matrix that projects vectors onto the column-space of $\mathbf{H}$.

As pointed out in [23,14], the cost function in (7) is a non-convex function. Thus, its maximization requires a multidimensional search: The space of possible source locations $\mathcal{R}^p$ ($p \in \{2,3\}$) is made discrete by considering q^p points that form a hyper-cube with q points along each dimension. Each one of the K sources is then assumed to be located at one of these points, resulting in a total of q^{pK} possible cases. For each one of these cases, the corresponding matrix $\mathbf{P_H}$ must be computed so as to evaluate the cost function in (8). In the case where two or more sources are assumed to be located at the same grid point, the number of columns of matrix $\mathbf{H}$ must be reduced accordingly so as to match the number of non-collocated sources, in order to ensure that matrix $\mathbf{H}^T\mathbf{H}$ is non-singular. In [23], several alternative optimization methods such as the multi-resolution (MR) search and the expectation-maximization (EM) were also studied.

3.2. The AP estimator

The alternating projection is a conceptually simple technique for multidimensional maximization. This technique has been successfully used in the past for multiple source localization using DOA measurements [27] and TDOA measurements of wideband signals in the near field [28]. In this subsection we adapt this approach to the case of multiple source localization using RSS measurements. The technique is iterative; at every iteration a maximization is performed with respect to a single parameter while all the other parameters remain fixed. In the case of multiple source localization, at each iteration, the cost function is optimized with respect to the location vector of a single source while the location vectors of all the other sources remain fixed. Thus, instead of q^{pK} evaluations of the cost function that are required for ML estimation, the alternating projection approach requires Kq^p evaluations to estimate the location of the K sources. Usually, the aforementioned iterative procedure may be executed M times, in this case, each source location estimate is "refined" while the estimates of the remaining $K-1$ sources remain equal to their previous values and MKq^p evaluations of the cost function are required. Of course, similarly to the ML estimation procedure, alternative optimization techniques such as the multi-resolution search or the EM algorithm may be used for the one-dimensional optimization steps of the AP algorithm. Furthermore, the projection matrix update formula [27] may be used to obtain a low-complexity implementation, as will be described in the sequel.

Consider that after an initialization phase, we have obtained the estimates of the source locations denoted as $\hat{\mathbf{x}}_1^{(0)}, \hat{\mathbf{x}}_2^{(0)}, \ldots, \hat{\mathbf{x}}_K^{(0)}$ and we wish to update the location estimate of the first source from $\hat{\mathbf{x}}_1^{(0)}$ to $\hat{\mathbf{x}}_1^{(1)}$. According to the AP approach, we must maximize (8) under the constraint that only the first column of matrix $\mathbf{H}$, involved in $\mathbf{P_H}$, is allowed to vary. Let us denote this matrix as

$$\mathbf{H}_1^{(1)} = [\mathbf{h}(\mathbf{x}_1)\ \ \mathbf{h}(\hat{\mathbf{x}}_2^{(0)})\ \ \cdots\ \ \mathbf{h}(\hat{\mathbf{x}}_K^{(0)})]. \tag{9}$$

The corresponding matrix that projects onto the columns space of $\mathbf{H}_1^{(1)}$ will be given by

$$\mathbf{P}_1^{(1)} = \mathbf{H}_1^{(1)}(\mathbf{H}_1^{(1)T}\mathbf{H}_1^{(1)})^{-1}\mathbf{H}_1^{(1)T}. \tag{10}$$

Now, the projection matrix update formula can be used to avoid the matrix inversion required for the computation of matrix $\mathbf{P}_1^{(1)}$. Let $\mathbf{B}$ and $\mathbf{C}$ be two arbitrary matrices with the same number of rows, and let $\mathbf{P}_{[\mathbf{B}|\mathbf{C}]}$ denote the projection matrix onto the columns space of the augmented matrix $[\mathbf{B}|\mathbf{C}]$. According to the projection matrix update formula, $\mathbf{P}_{[\mathbf{B}|\mathbf{C}]}$ can be expressed as the sum of the projection matrix $\mathbf{P_C}$ and the projection matrix $\mathbf{P}_{\mathbf{B_C}}$, where $\mathbf{B_C}$ denotes the residual of the columns of $\mathbf{B}$ when projected on $\mathbf{C}$:

$$\mathbf{P}_{[\mathbf{B}|\mathbf{C}]} = \mathbf{P_C} + \mathbf{P}_{\mathbf{B_C}}. \tag{11}$$

Matrix $\mathbf{B_C}$ can be written as

$$\mathbf{B_C} = (\mathbf{I} - \mathbf{P_C})\mathbf{B}. \tag{12}$$

Thus, matrix $\mathbf{P}_1^{(1)}$ can be computed in K steps, where at each one such step we consider a matrix augmented with a column vector $\mathbf{B} = \mathbf{h}(\hat{\mathbf{x}}_k)$ from matrix $\mathbf{H}_1^{(1)}$. Since matrix $\mathbf{B}$ is a vector, the required matrix $\mathbf{P}_{\mathbf{B_C}}$ can be computed without the need to compute any matrix inversion. In particular, we compute $\mathbf{P}_1^{(1)}$ using a sequence of $K+1$ matrices $\mathbf{P}_1^{(1,0)} = \mathbf{0}_{N_a}$, $\mathbf{P}_1^{(1,1)}, \ldots, \mathbf{P}_1^{(1,K)} = \mathbf{P}_1^{(1)}$ by application of the recursive formula

$$\mathbf{P}_1^{(1,k+1)} = \mathbf{P}_1^{(1,k)} + \frac{(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,k)})\mathbf{h}_k\mathbf{h}_k^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,k)})}{\mathbf{h}_k^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,k)})\mathbf{h}_k}, \tag{13}$$

where $\mathbf{h}_k$ denotes a column of matrix $\mathbf{H}_1^{(1)}$ and the sequence of columns can be arbitrary. Now, considering that the last update uses the first column of $\mathbf{H}_1^{(1)}$, we have that

$$\mathbf{P}_1^{(1)} = \mathbf{P}_1^{(1,K-1)} + \frac{(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})\mathbf{h}(\mathbf{x}_1)\mathbf{h}(\mathbf{x}_1)^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})}{\mathbf{h}(\mathbf{x}_1)^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})\mathbf{h}(\mathbf{x}_1)}. \tag{14}$$

If we substitute $\mathbf{P}_1^{(1)}$ from (14) in (8), we obtain two terms. We notice that the first term, $\mathbf{y}^T\mathbf{P}_1^{(1,K-1)}\mathbf{y}$, does not depend on $\mathbf{x}_1$. Thus, we have that the maximization of (8) with respect to $\mathbf{x}_1$ (and keeping all the other source locations constant) is equivalent to the following maximization:

$$\hat{\mathbf{x}}_1^{(1)} = \underset{\mathbf{x}_1}{\operatorname{argmax}} \left(\frac{\mathbf{y}^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})\mathbf{h}(\mathbf{x}_1)\mathbf{h}(\mathbf{x}_1)^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})\mathbf{y}}{\mathbf{h}(\mathbf{x}_1)^T(\mathbf{I}_{N_a} - \mathbf{P}_1^{(1,K-1)})\mathbf{h}(\mathbf{x}_1)} \right). \tag{15}$$

In general, when the alternating projection is executed at iteration m ($m = 1,2,\ldots,M$), the estimation of the location vector of the k-th ($k = 1,2,\ldots,K$) source must be selected as the vector that maximizes the function

$$J(\mathbf{x}_k) = \left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a} - \mathbf{P}_k^{(m,K-1)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a} - \mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a} - \mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)} \right), \tag{16}$$

where $\mathbf{P}_k^{(m,K-1)}$ is the projection matrix onto the column space of matrix

$$\mathbf{H}_k^{(m,K-1)} = [\mathbf{h}(\hat{\mathbf{x}}_1^{(m)})\ \ \ldots\ \ h(\hat{\mathbf{x}}_{k-1}^{(m)})\ \ \mathbf{h}(\hat{\mathbf{x}}_{k+1}^{(m-1)})\ \ \ldots\ \ \mathbf{h}(\hat{\mathbf{x}}_K^{(m-1)})], \tag{17}$$

which has $K-1$ columns, defined by the respective $\mathbf{h}(\cdot)$ vectors of the current estimates about the locations of the remaining $K-1$ sources.

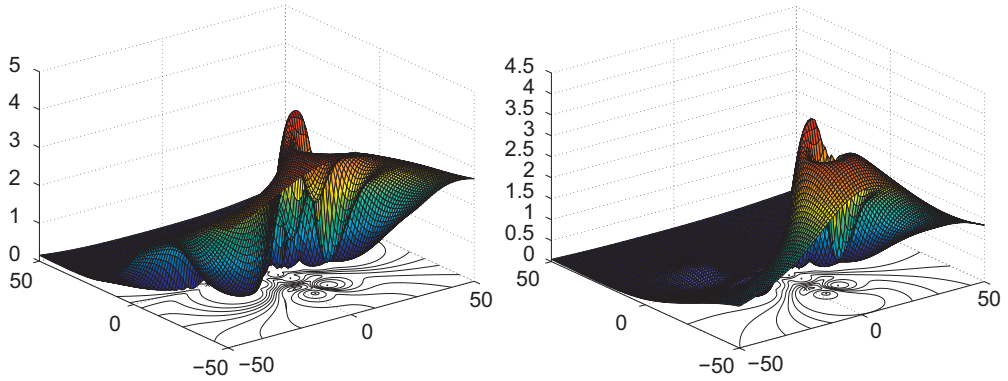

Fig. 1. (a) A plot of the function in (16), where there are 25 active nodes and 4 sources. The first three sources are at $(20,0)$, $(0,20)$, $(-20,0)$ and have been estimated correctly, while the plot depicts the function that must be maximized for estimating the fourth source at $(0,-20)$. (b) The respective plot for the approximate function in (21).

3.3. Complexity analysis of the AP scheme

As it is clear from the previous, the update of a source location from $\hat{\mathbf{x}}_k^{(m-1)}$ to $\hat{\mathbf{x}}_k^{(m)}$ requires the construction of matrix $\mathbf{P}_k^{(m,K-1)}$ and the maximization of the respective function in (16). The computation of $\mathbf{P}_k^{(m,K-1)}$ is performed in $K-1$ steps, where at each such step the most computationally demanding operation is a matrix–vector product, hence, this computation requires $O((K-1)N_a^2)$ operations. On the other hand, maximization of (16) must be performed using some search-based method. This is due to the fact that this function is non-convex, as it is demonstrated in Fig. 1. Each evaluation of (16) requires $O(N_a^2)$ operations, for computing the matrix–vector product $(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)$. Thus, the evaluation of (16) at q^p points would require $O(q^p\cdot N_a^2)$ operations. In total, M iterations with K sources and the use of a grid of q^p points would require $O(M\cdot K((K-1)+q^p)N_a^2)$ operations. Clearly, the dominant part in the complexity of the method is due to the maximization operation, since q^p is typically much greater than $K-1$.

4. Low complexity approximate AP

4.1. Motivation

In the previous, it was shown that the alternating projection method requires the maximization of functions of the form in (16). Also, the complexity of evaluating functions of this form was shown to be proportional to the square of the number of nodes N_a that participate in the estimation. On the other hand, the function that must be maximized for performing ML estimation in the single source case is given by (8) as

$$J_{ML}(\mathbf{x})=\frac{\mathbf{h}(\mathbf{x})^T\mathbf{y}\mathbf{y}^T\mathbf{h}(\mathbf{x})}{\mathbf{h}(\mathbf{x})^T\mathbf{h}(\mathbf{x})}, \tag{18}$$

where we have considered that $\mathbf{H}$ in (8) is equal to a vector $\mathbf{h}(\mathbf{x})$. Clearly, the above function can be evaluated using only $O(N_a)$ operations, since only vector-by-vector multiplication is involved. This fact motivates us to consider if it is possible to device a similar cost function that could be used for performing the single-source localization operations involved in the AP scheme. Such an approach could reduce the overall complexity of the AP method from $O(M\cdot K((K-1)+q^p)N_a^2)$ to $O(M\cdot K((K-1)N_a^2+q^p\cdot N_a))$. As an example, in a case where $M=6, K=3$ and $N_a=10$, this would reduce the complexity due to evaluation of the function, from $1800\cdot q^p$ to $180\cdot q^p$.

4.2. An approximation to the cost function

If we compare the function in (18) with the function in (16), we notice that they have a similar form. In particular,

(1) The numerator in (16) can be written in the form of the numerator in (18) by defining vector $\mathbf{y}_C^{(m,k)}=(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{y}$. Furthermore, it is easy to verify that $\mathbf{y}_C^{(m,k)}$ represents a vector of "canceled" measurements. This can be verified by considering that

$$\begin{aligned}\mathbf{y}_C^{(m,k)}&=(\mathbf{I}-\mathbf{P}_k^{(m,K-1)})\mathbf{y}\\&=\mathbf{y}-\mathbf{P}_k^{(m,K-1)}\mathbf{y}\\&=\mathbf{y}-\mathbf{H}_k^{(m,K-1)}(\mathbf{H}_k^{(m,K-1)^T}\mathbf{H}_k^{(m,K-1)})^{-1}\mathbf{H}_k^{(m,K-1)^T}\mathbf{y}\\&=\mathbf{y}-\mathbf{H}_k^{(m,K-1)}\hat{\alpha}^{(m,k)},\end{aligned} \tag{19}$$

where

$$\hat{\alpha}^{(m,k)}=(\mathbf{H}_k^{(m,K-1)^T}\mathbf{H}_k^{(m,K-1)})^{-1}\mathbf{H}_k^{(m,K-1)^T}\mathbf{y} \tag{20}$$

represents the least squares estimate for the powers of the remaining $K-1$ sources, as it can be seen by comparing it with Eq. (6). Thus, the product $\mathbf{H}_k^{(m,K-1)}\hat{\alpha}^{(m,k)}$ is an estimate of the energy measurements that would be obtained in the case where only the remaining $K-1$ sources were present, and vector $\mathbf{y}_C^{(m,k)}$ denotes the measurements after removal of their contribution.

(2) The denominator in (16) is equal to the denominator in (18) apart from the term $\mathbf{h}(\mathbf{x}_k)^T\mathbf{P}_k^{(m,K-1)}\mathbf{h}(\mathbf{x}_k)$. Thus, we conclude that this term represents a correction due to "interference" caused by the presence of the other $K-1$ sources that cannot be taken into account

by means of the canceled measurements $\mathbf{y}_C^{(m,k)}$. However, when $\mathbf{h}(\mathbf{x}_k)^T\mathbf{h}(\mathbf{x}_k)$ is large as compared to $\mathbf{h}(\mathbf{x}_k)^T\mathbf{P}_k^{(m,K-1)}\mathbf{h}(\mathbf{x}_k)$, then the last term should have only a small contribution to the evaluation of (16).

Based upon the above discussion, we define the function

$$I(\mathbf{x}_k)=\left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T\mathbf{h}(\mathbf{x}_k)}\right) \tag{21}$$

as an approximation of the function in (16), when $\mathbf{h}(\mathbf{x}_k)^T\mathbf{h}(\mathbf{x}_k)$ is large as compared to $\mathbf{h}(\mathbf{x}_k)^T\mathbf{P}_k^{(m,K-1)}\mathbf{h}(\mathbf{x}_k)$. Thus, we suggest using a threshold λ, and test if

$$\frac{\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})^T\mathbf{P}_k^{(m,K-1)}\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})}{\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})^T\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})}<\lambda, \tag{22}$$

that is, test if $I(\mathbf{x}_k)$ is a good approximation of the correct function $J(\mathbf{x}_k)$, close to the previous estimate $\hat{\mathbf{x}}_k^{(m-1)}$. In this case, $I(\mathbf{x}_k)$ can be used in the optimization operation. Fig. 1 demonstrates that this function is also non-convex. Considering that we need to evaluate (21) at q^p points, we can first compute the canceled measurements vector $\mathbf{y}_C^{(m,k)}$ in N_a^2 operations, and then use this vector to maximize (21) by performing $O(N_a\cdot q^p)$ operations. Of course, the overall complexity of the proposed AP-based algorithm will depend on the outcome of the suggested function-selection criterion. The parameter λ, ranging in the interval [0,1], can be used as a trade-off between complexity and accuracy. The proposed AP-based multiple source localization algorithm is summarized in Table 1.

Since the examined algorithm is based upon the so-called "deterministic hill climbing" method, its convergence behavior is difficult to be analyzed. Furthermore, one of the most important factors for convergence is the selection of the initial estimates. In the next subsection we propose an initialization method that, by means of simulations, was found to lead to accurate initial estimates of the locations of the sources.

4.3. Suggested initialization procedure

In the previous, we assumed that both the exact AP and the low-complexity approximate AP-based algorithms were supplied with initial estimates $\hat{\mathbf{x}}_1^{(0)}$, $\hat{\mathbf{x}}_2^{(0)},\ldots,\hat{\mathbf{x}}_K^{(0)}$ of the locations of the sources. If we adopt the method proposed in [27] into our case, we arrive at the algorithm of Table 2. In this paragraph, we propose a two-stage multi-resolution method for performing the function maximization operations that appear in Table 2.

At the first stage of the multi-resolution search, we propose to use a coarse grid that coincides with the locations of active sensors. Using this approach, we have that vector $\mathbf{h}(\mathbf{r}_i)/\|\mathbf{h}(\mathbf{r}_i)\|$ will be equal to the i-th vector of the N_a- dimensional canonical basis, that is

$$\frac{\mathbf{h}(\mathbf{r}_i)}{\|\mathbf{h}(\mathbf{r}_i)\|}=\mathbf{e}_i=[\underbrace{0\ 0\ \cdots\ 0}_{i-1\text{ zeros}}1\underbrace{0\ 0\ \cdots\ 0}_{N_a-i\text{ zeros}}]^T. \tag{23}$$

Thus, the problem that must be solved can be written as

$$\hat{\mathbf{x}}_k^{(-1)}=\arg\max_{\mathbf{x}_k,\ \mathbf{h}(\mathbf{x}_k)\in\{\mathbf{e}_1,\ldots,\mathbf{e}_{N_a}\}}\left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\mathbf{x}_k)}\right) \tag{24}$$

and its solution involves computation of the vector $(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{y}$, squaring each element, division of each element by the respective diagonal element of matrix $\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)}$, and finally selection of the maximum value. Thus, this optimization step can be performed in $O(N_a^2)$ operations.

For the second stage of the multi-resolution search, a finer grid must be constructed close to the location of the node that was selected at the previous stage. Thus, a

Table 1
The proposed approximate AP-based multiple source localization algorithm.

INPUT: Initial location estimates $\hat{\mathbf{x}}_1^{(0)},\hat{\mathbf{x}}_2^{(0)},\ldots,\hat{\mathbf{x}}_K^{(0)}$, measurements $\mathbf{y}$
OUTPUT: Final location estimates after M iterations: $\hat{\mathbf{x}}_1^{(M)},\hat{\mathbf{x}}_2^{(M)},\ldots,\hat{\mathbf{x}}_K^{(M)}$

FOR $m=1$ TO M
  FOR $k=1$ TO K
    $\mathbf{P}_k^{(m,0)}=\mathbf{0}_{N_a}$
    FOR $i=1$ TO $k-1$
      $\mathbf{P}_k^{(m,i)}=\mathbf{P}_k^{(m,i-1)}+\frac{(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-1)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-1)})}{\mathbf{h}(\hat{\mathbf{x}}_i^{(m)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-1)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m)})}$
    END
    FOR $i=k+1$ TO K
      $\mathbf{P}_k^{(m,i-1)}=\mathbf{P}_k^{(m,i-2)}+\frac{(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-2)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m-1)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m-1)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-2)})}{\mathbf{h}(\hat{\mathbf{x}}_i^{(m-1)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,i-2)})\mathbf{h}(\hat{\mathbf{x}}_i^{(m-1)})}$
    END
    IF $\frac{\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})^T\mathbf{P}_k^{(m,K-1)}\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})}{\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})^T\mathbf{h}(\hat{\mathbf{x}}_k^{(m-1)})}<\lambda$,
      $\hat{\mathbf{x}}_k^{(m)}=\operatorname{argmax}_{\mathbf{x}_k}\left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T\mathbf{h}(\mathbf{x}_k)}\right)$
    ELSE
      $\hat{\mathbf{x}}_k^{(m)}=\operatorname{argmax}_{\mathbf{x}_k}\left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(m,K-1)})\mathbf{h}(\mathbf{x}_k)}\right)$
    END
  END
END

Table 2
Initialization phase of the EB-AP-MSL algorithm.

INPUT: Measurements $\mathbf{y}$
OUTPUT: Initial location estimates $\hat{\mathbf{x}}_1^{(0)},\hat{\mathbf{x}}_2^{(0)},\ldots,\hat{\mathbf{x}}_K^{(0)}$

$\mathbf{P}_1^{(0)}=\mathbf{0}_{N_a}$
FOR $k=1$ TO $K-1$
  $\hat{\mathbf{x}}_k^{(0)}=\operatorname{argmax}_{\mathbf{x}_k}\left(\frac{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\mathbf{x}_k)}{\mathbf{h}(\mathbf{x}_k)^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\mathbf{x}_k)}\right)$
  $\mathbf{P}_{k+1}^{(0)}=\mathbf{P}_k^{(0)}+\frac{(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\hat{\mathbf{x}}_k^{(0)})\mathbf{h}(\hat{\mathbf{x}}_k^{(0)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})}{\mathbf{h}(\hat{\mathbf{x}}_k^{(0)})^T(\mathbf{I}_{N_a}-\mathbf{P}_k^{(0)})\mathbf{h}(\hat{\mathbf{x}}_k^{(0)})}$
END
$\hat{\mathbf{x}}_K^{(0)}=\operatorname{argmax}_{\mathbf{x}_K}\left(\frac{\mathbf{h}(\mathbf{x}_K)^T(\mathbf{I}_{N_a}-\mathbf{P}_K^{(0)})\mathbf{y}\mathbf{y}^T(\mathbf{I}_{N_a}-\mathbf{P}_K^{(0)})\mathbf{h}(\mathbf{x}_K)}{\mathbf{h}(\mathbf{x}_K)^T(\mathbf{I}_{N_a}-\mathbf{P}_K^{(0)})\mathbf{h}(\mathbf{x}_K)}\right)$

problem that we must also investigate has to do with the size of the grid. To this end, we view the estimate of the previous stage of the search as being an approximation of the closest point of approach (CPA) node, for the k-th source that we wish to estimate. This interpretation will help us select a proper size for the fine search grid. In addition, in the following we adopt a stochastic model for the locations of the nodes of the sensor network, and in particular we assume that their locations can be assumed random and uniformly distributed over the region of interest.

In the following, we will derive the expected distance between a source and its CPA node, in the two dimensional and three dimensional cases. Let us consider that n sensor nodes are uniformly deployed over a region of area E. We may define the *spatial density* of such a network as $d = n/E$. Consider also that a source is placed in the same region, and define a circular area R of radius ρ around that source. Define also the random variable Y_{2D} denoting the distance between the source and the closest sensor node. Then, it follows that the probability that the closest sensor node lies in R is given by

$$\begin{aligned}\Pr\{Y_{2D}<\rho\} &= \Pr\{(\text{number of nodes in } R)\geq 1\}\\ &= 1-\Pr\{(\text{number of nodes in } R)=0\}\\ &= 1-\left(1-\pi d\rho^2\frac{1}{n}\right)^n. \end{aligned} \tag{25}$$

Thus, as n and $E = n/d$ tend to infinity, we have that

$$\Pr\{Y_{2D}<\rho\} = 1-e^{-\pi d\rho^2}. \tag{26}$$

Differentiating the above cumulative density with respect to ρ, we get the probability density function of Y_{2D} as

$$f_{Y_{2D}}(\rho) = 2\pi\, d\rho e^{-\pi d\rho^2}. \tag{27}$$

The expected value of Y_{2D}, which is the expected distance of the source from its CPA node, is thus found to be

$$\mu_{2D} = E[Y_{2D}] = \int_0^{+\infty}\rho\cdot f_{Y_{2D}}(\rho)\,d\rho = \frac{1}{2\sqrt{d}}. \tag{28}$$

Using similar reasoning for the three dimensional case and defining the random variable Y_{3D} to represent the distance between the source and the closest node, we have that

$$\Pr\{Y_{3D}<\rho\} = 1-\left(1-\frac{4}{3}\pi\, d\rho^3\frac{1}{n}\right)^n, \tag{29}$$

and the respective probability density function becomes

$$f_{Y_{3D}}(\rho) = 4\pi d\rho^2 e^{-(4/3)\pi d\rho^3}. \tag{30}$$

Finally, the expected value of the distance becomes

$$\mu_{3D} = E[Y_{3D}] = \int_0^{+\infty}\rho\cdot f_{Y_{3D}}(\rho)\,d\rho = \frac{\Gamma(1/3)}{\sqrt[3]{36\pi d}}. \tag{31}$$

Thus, taking into account that the estimates $\hat{\mathbf{x}}_k^{(-1)}$ can be interpreted as the CPA nodes, we propose to consider a rectangular region of size $4\mu_{2D}\times 4\mu_{2D}$ for $p=2$, and a region of size $4\mu_{3D}\times 4\mu_{3D}\times 4\mu_{3D}$ for $p=3$. Within these regions, we must construct a $q\times q$ grid or a $q\times q\times q$ grid of points, respectively, centered at the location $\hat{\mathbf{x}}_k^{(-1)}$. The probability that the source is located within a circle (respectively, sphere), of radius $2\mu_{2D}$ (respectively, $2\mu_{3D}$) centered at the actual CPA node, is independent of the density d of the network, and was evaluated to be equal to 0.95 (respectively, 0.99). In the case where the assumption of uniform deployment does not hold, or the density of the network is not known, then a "local" estimate of the density can be obtained by considering a number of nodes in the vicinity of $\hat{\mathbf{x}}_k^{(-1)}$ as well as the area or volume they occupy. Such an approach requires the assumption of uniform deployment to hold only locally.

4.4. Distributed implementation issues

The localization method proposed in the previous can easily be implemented in a centralized fashion. In particular, the information that must be transmitted to a node, so-called *fusion-center*, comprises of the measurements and locations of active nodes, giving a total of $(p+1)\cdot N_a$ real numbers, assuming that each node can communicate with the fusion-center directly. On the other hand, in a distributed implementation of the algorithm the following facts may be taken into account:

(1) From the previous section, we have that the projection matrices involved in the localization algorithm depend on the locations of active sensor nodes and the current source location estimates. Thus, this information must be available at a sensor node, so that a local copy of the associated projection matrix can be computed. To this end, at the end of every location-update step, the derived estimate must be transmitted to all active nodes. This requires a total of $(M+1)\cdot K\cdot p\cdot(N_a-1)$ real numbers. We may exclude the transmission of the location vectors of the active nodes, assuming that they have been transmitted during an initialization phase and they remain the same (i.e. the nodes do not move).

(2) Of course, the most demanding operation that must be implemented has to do with the evaluation of the cost function. For this task, we first note that if all active nodes know the current locations of the sources, then each node can generate a local copy of the search grid, according to a predefined rule. Thus, each node can generate the required vector $\mathbf{h}(\mathbf{x})$ for a test point $\mathbf{x}$ of the grid. In the sequel, the active nodes must cooperate to compute a term of the form $\mathbf{y}^T(\mathbf{I}-\mathbf{P})\mathbf{h}(\mathbf{x})$. To this end, each node i must compute the i-th element v_i of the vector $(\mathbf{I}-\mathbf{P})\mathbf{h}(\mathbf{x})$, and multiply it with the local measurement y_i. Then, the sum of the products $v_i\cdot y_i$ must be computed. This will require the transmission of N_a-1 real numbers, assuming that N_a-1 nodes send their contribution to some "leader node", for example the node estimated as the closest point of approach for each source. Finally, this leader node will compute the sum of the contributions, square it, and divide the result by either $\mathbf{h}^T(\mathbf{x})(\mathbf{I}-\mathbf{P})\mathbf{h}(\mathbf{x})$ or $\mathbf{h}^T(\mathbf{x})\mathbf{h}(\mathbf{x})$, according to whether the exact or the approximate cost function is used. In total, evaluation of the cost functions will require the transmission of $(M+1)\cdot K\cdot q^p\cdot(N_a-1)$ real numbers.

As it becomes clear from the above, the examined distributed implementation requires much more communication as compared to the centralized one. Thus, we draw the conclusion that it is preferred to implement the proposed localization algorithms in a centralized fashion.

5. Simulation results

In order to evaluate the performance of the proposed localization algorithms, we conducted some typical numerical simulations. In the following subsections, we present the results of these experiments categorized as follows: (a) In Section 5.1, we compare the performance of the AP-based algorithms with that of the ML estimator, in a scenario where two sources must be localized. (b) The performance of the AP-based algorithms in localizing three equidistant sources is demonstrated in Section 5.2. (c) The performance of the proposed algorithms, under various perturbations in the model, is studied in Section 5.3.

In a localization problem involving multiple sources, there are $K\cdot(p+1)$ unknowns, namely the coordinates and powers of the K sources. Thus, the number of active sensors should at least be equal to that number. As an example, for $K=2$ sources in a two dimensional scenario, this number is equal to 6. Thus, in the experiments presented in the following, we have included only instances of the localization problem in which a minimum number of active nodes exists.

Also, the problem of data association [1] must be taken into account. In our setting, the problem is to associate the K location estimates of the localization algorithm, with the actual locations of the K sources, in order to measure the respective location errors. In all the following experiments, data association is performed by computing all possible $K!$ possible associations, and choosing the one that results in the smallest sum of square errors. Furthermore, after data association, we propose to differentiate the sources in the sense that we number them according to their respective location error, i.e. the source estimated with the smallest error is considered as the first one and so on. As it will become more clear in the following, such a sorting of the estimated sources allows us to better interpret the performance characteristics of multiple source localization algorithms.

5.1. Localization performance for two sources

In order to compare the performance of the proposed algorithm to that of the ML estimator, we simulated several realizations of the multiple source localization problem. At each instance, $N=500$ sensor nodes were uniformly deployed over a $p=2$ dimensional $100\times 100\,\mathrm{m}^2$ field. In the same field two sources of equal strength $A_1=A_2=100$ were also placed. In particular, in order to avoid boundary effects (i.e. instances where the sources lie close to the boundaries of the sensor field), we uniformly placed the first source at $\mathbf{x}_1$ in the central $50\times 50\,\mathrm{m}^2$ field, and the second source was placed at $\mathbf{x}_2=\mathbf{x}_1-\delta\mathbf{x}_1/\|\mathbf{x}_1\|$, so that both sources always lie in the central 50×50 field if their distance δ is smaller than 25 m (the origin point $(0,0)$ was the center of the square field). The measurements of the sensors were generated according to the model of Eq. (1), the variance of the noise was set to $\sigma^2=1$, and active nodes were detected using a threshold $T=5$. To avoid ill-conditioned realizations, only the realizations that resulted in $N_a>9$ nodes were simulated. In order to test how the examined algorithms perform with respect to the distance between the two sources, we varied δ from 2.5 to 25 m with a step size of 2.5 m. For each case, we simulated 2000 realizations of the multiple source localization problem and computed the respective average errors.

For the AP and the low-complexity AP-based algorithms, we performed seven iterations (i.e. $M=6$), using the initialization procedure of Section 4.3. At the initialization ($M=0$), a square 31×31 ($q=31$) grid was used, extended to an area $4/(2\sqrt{d})\times 4/(2\sqrt{d})\,\mathrm{m}^2$ as suggested in Section 4.3, where the network density is $d=500/(100\times 100)$. At the following iterations ($m>0$), we also used 31×31 square grids, measuring $1/(2^{m-1}\sqrt{d})\times 1/(2^{m-1}\sqrt{d})$ m. The centers of these grids were the source location estimates of the previous iteration.

For the ML estimator, we used the multi-resolution [23] approach with the same size grids (size of region and number of points) as for the AP-based algorithms and also seven iterations. In order to reduce experimentation time for the computationally demanding ML estimator, the centers of the initial grids were placed at the correct closest point of approach nodes of each source, assuming that a very detailed initial search procedure would eventually detect these regions.

In Figs. 2(a) and (b) the root mean square (RMS) and root median square errors of the examined algorithms are given, respectively, as a function of the distance δ between the two sources. From these figures, we note that the proposed low-complexity AP-based algorithm with $\lambda=0.05$ obtains very similar performance to the exact AP implementation. In terms of root median square error, a performance difference is evident only in the interval from 5 to 12.5 m, where the exact implementation performs better by at most 0.2 m. Also, the AP-based algorithms obtain a root median square error that is comparable to that of the ML estimator, especially when $\delta\geq 15$ m where the ML estimate is at most 0.1 m better.

In Fig. 2(c) we demonstrate the root median square error for the low-complexity AP-based algorithm and three different values for λ. Also, in Fig. 2(d), the respective percentages of choosing the approximate functions for maximization are presented. From these figures, the trade-off between performance and complexity is presented: Choosing a value for λ close to zero, results in a performance close to that of the exact implementation, but this results in a reduction of the percentage of choosing the approximate functions. However, for $\delta\geq 15$ m, even the value of $\lambda=0.95$ results in performance equivalent to that of the exact implementation.

In Fig. 2(e), we demonstrate the average numbers of active nodes for the simulated realizations. In total, 11.3

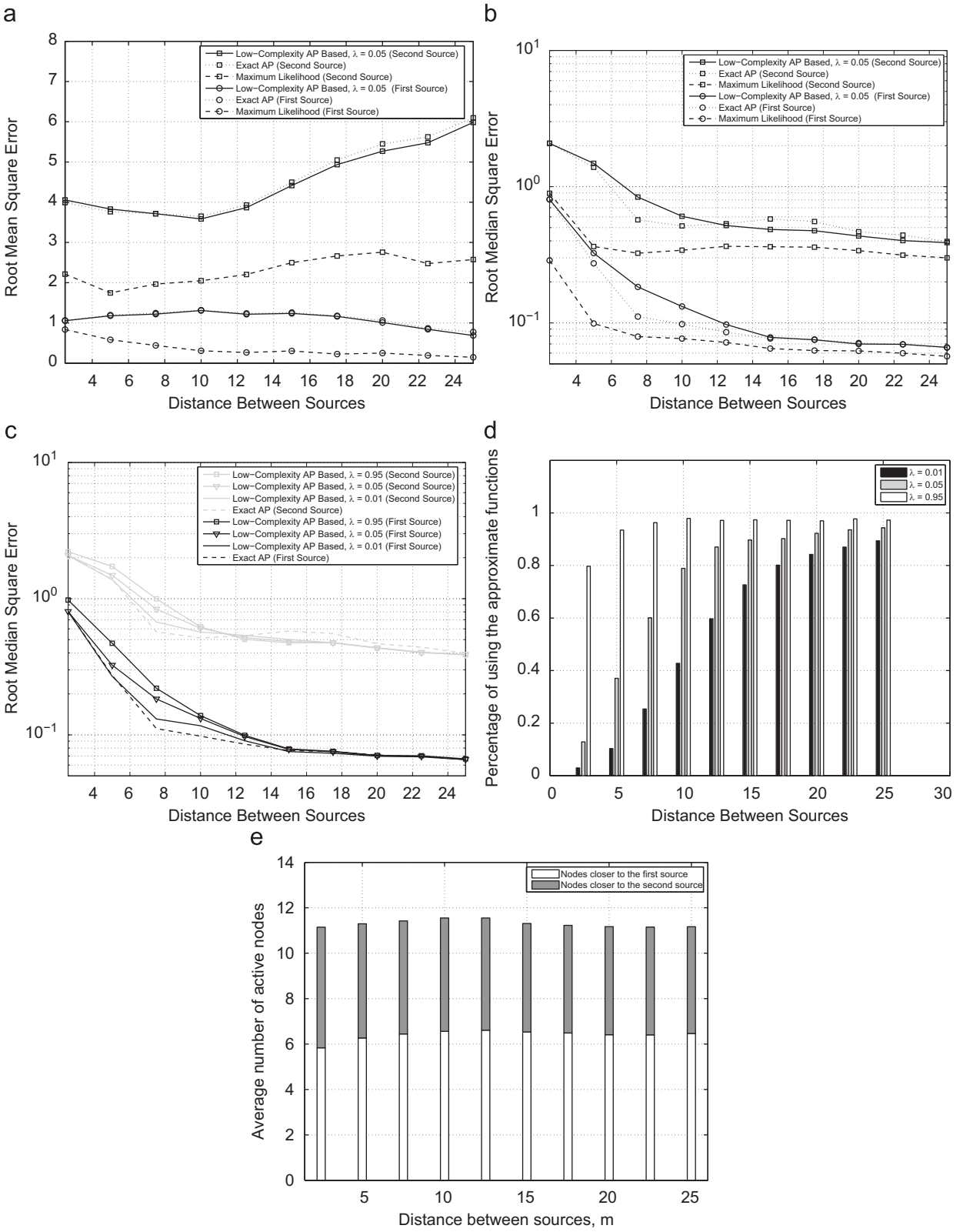

Fig. 2. (a) RMS localization error for two sources, (b) root median square error for two sources, (c) root median square error for the low complexity AP-based algorithm and various λ, (d) respective percentages of using the approximate functions, (e) average number of active sensor nodes.

nodes were on average active, 6.4 of which were closer to the better estimated source and 4.9 closer to the second source. The fact that the number of active nodes closer to the second source is small, results in an RMS error for the second source that increases for $\delta > 12.5$, as shown in Fig. 2(a). In particular, while for $\delta < 12.5$ m all active nodes are close enough to both sources, as the distance between the sources increases, fewer active sensors are close to the second source, and thus ill-conditioned instances with less than 4 active nodes close to the second source become more probable. The root median square error depicted in Figs. 2(b) and (c) "filters" effectively these ill-conditioned realizations.

In Fig. 3, the root median square error of the AP-based algorithms is presented, as a function of the iteration m. We focus on the four cases $\delta = 5$ m and $N = 500$ nodes, $\delta = 10$ m and $N = 500$ nodes, $\delta = 5$ m and $N = 1000$ nodes and $\delta = 10$ m and $N = 1000$ nodes. From these figures we note that (a) a large value of λ may cause the algorithm to be trapped at some local maximum, (b) as the network density increases or the distance between the sources is increased, the performance gap between the ML and the AP-based algorithms becomes smaller, and (c) when the distance between the sources is large enough, even a large value for λ results in a performance very close to that of the exact AP scheme. The complexity gains, in terms of the percentages of times that the approximate functions were selected, appear in Table 3.

5.2. Localization performance for three sources

In this paragraph, we explore the performance of the proposed algorithm in localizing three sources. In particular, $N = 1000$ sensor nodes were uniformly deployed over a $p = 2$ dimensional $100 \times 100\,\text{m}^2$ field. In the same field three sources of equal strength $A_1 = A_2 = A_3 = 100$ were also placed on the vertices of an equilateral triangle

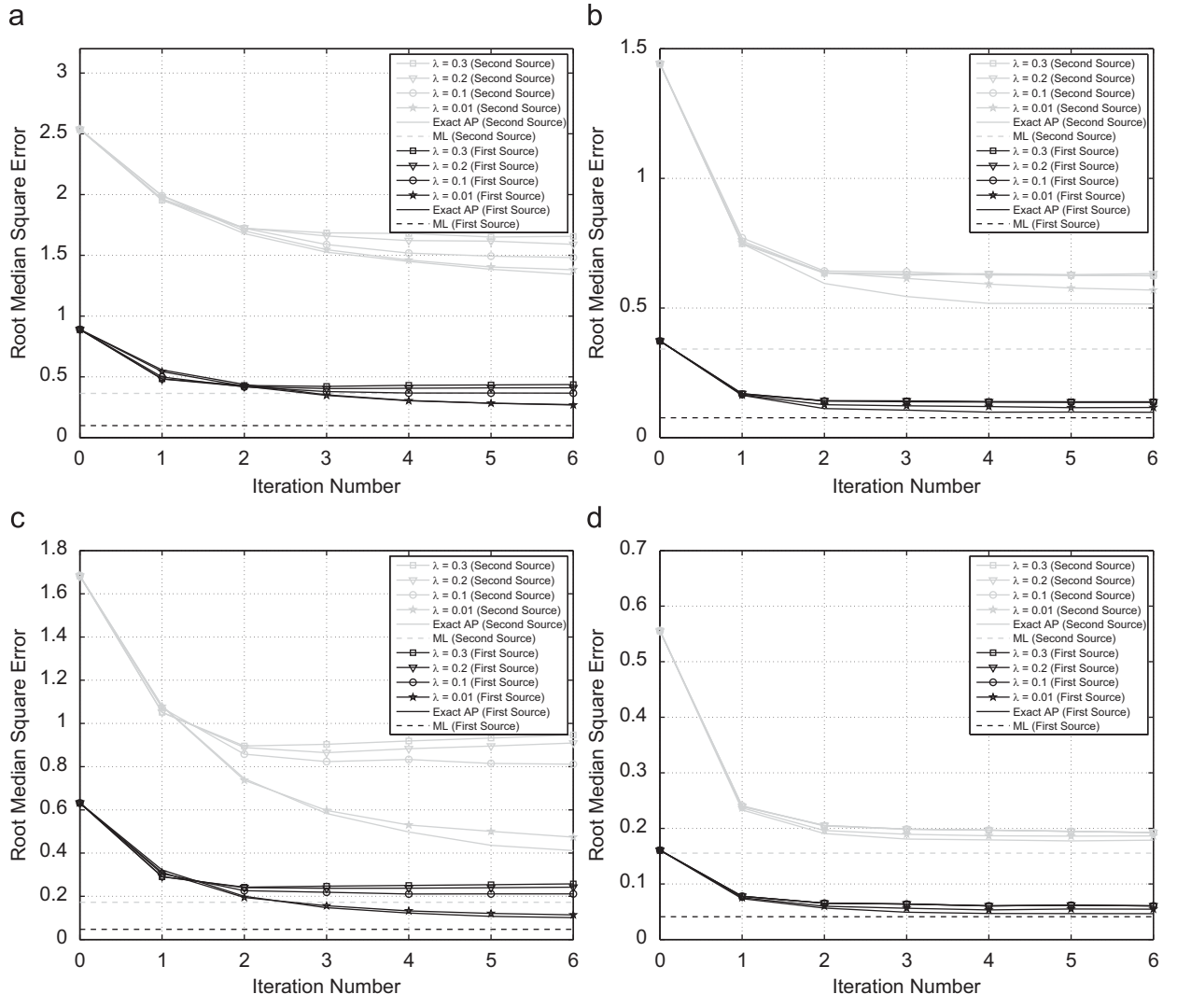

Fig. 3. Root median square error as a function of the iteration number (a) $\delta = 5$ m and density $d = 0.05$, (b) $\delta = 10$ m and density $d = 0.05$, (c) $\delta = 5$ m and density $d = 0.1$, (d) $\delta = 10$ m and density $d = 0.1$.

centered at the origin point $(0,0)$. All other parameters of the proposed algorithm are the same as those explained in the previous subsection. The distance δ between the sources was varied from 2.5 to 25 m in 2.5 m increments. For each δ, we simulated 2000 instances of the localization problem. In order to avoid ill conditioned instances, only those realizations of the problem that resulted in more than 14 active nodes were simulated. In Fig. 4 we demonstrate the resulting RMS and root median square errors for the locations of the three sources as well as the percentage of times that the low-complexity AP-based algorithm selected the approximate functions for $\lambda = 0.1$. These figures show the effectiveness of the proposed method for multiple source localization. In particular, the approximate scheme offers equivalent performance to the exact implementation, at a reduced computational complexity, as it can be seen from Fig. 4(c).

Table 3
Percentages of times that the approximate functions were selected, for the cases in Fig. 3.

	$N = 500$ nodes (%)		$N = 1000$ nodes (%)	
	$\delta = 5$ m	$\delta = 10$ m	$\delta = 5$ m	$\delta = 10$ m
$\lambda = 0.01$	10.87	43.96	16.36	58.03
$\lambda = 0.1$	53.05	88.37	64.96	94.36
$\lambda = 0.2$	69.07	93.36	79.12	96.40
$\lambda = 0.3$	77.45	95.19	86.48	96.56

5.3. Robustness to model perturbations

In this paragraph, we repeat the experiment of Section 5.1, but we now study the impact of possible inaccuracies in the model of Eq. (1). In particular, we study the case where the locations of the sensors are not known exactly,

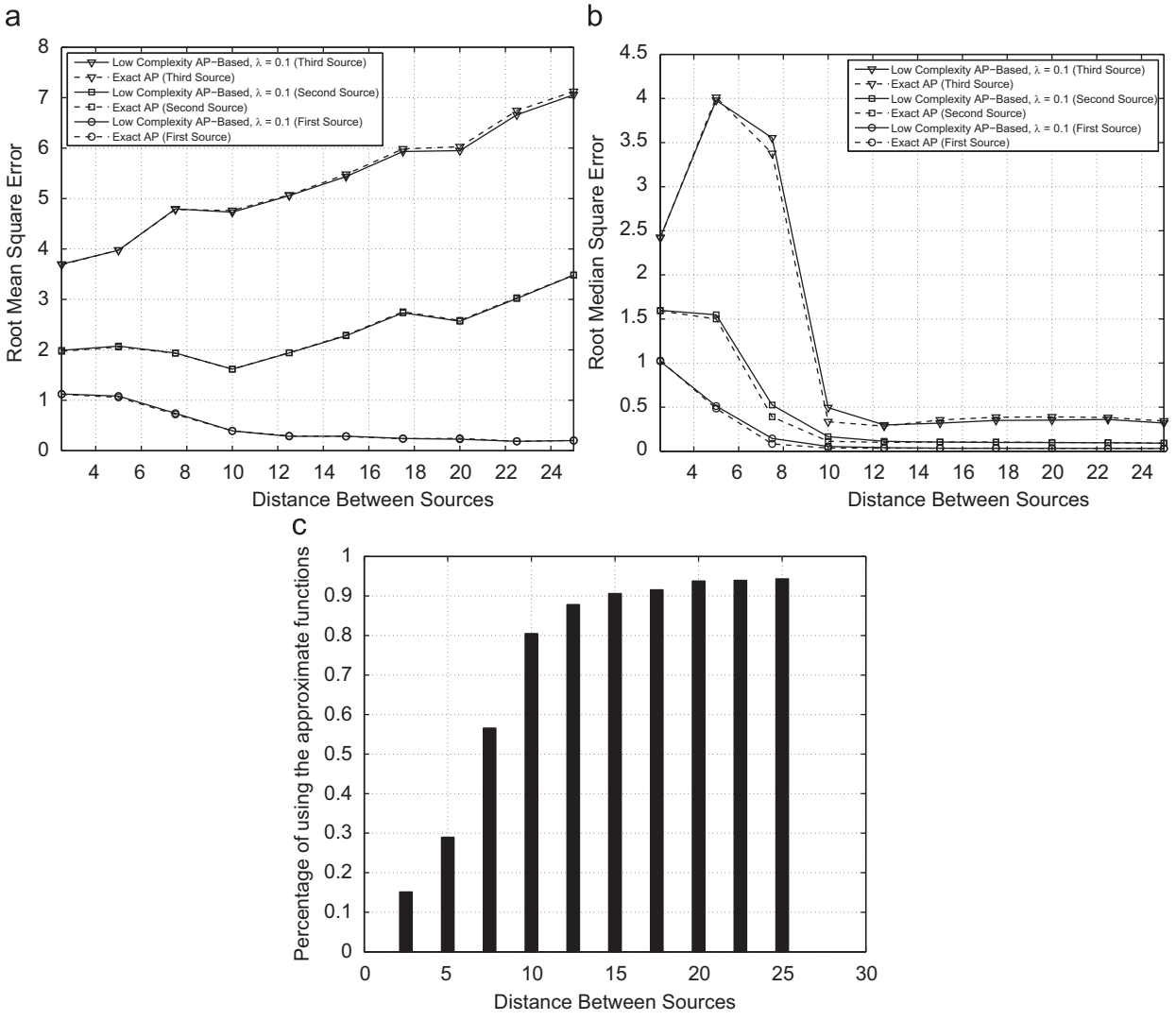

Fig. 4. (a) RMS localization error for the localization of three sources, (b) respective root median square error, (c) percentage of times of selecting the approximate functions.

but rather, each coordinate contains uniform error, and the case where the sensors are not well calibrated.

In Figs. 5(a) and (b), we demonstrate the root median square errors for the exact and approximate schemes, respectively, in the case where the coordinates of the sensors are perturbed by adding noise uniformly distributed in the interval $(-0.5, +0.5)$ meters. From these figures we note that there is a significant performance degradation due the erroneous estimates of the locations of the sensors. However, the exact and the approximate schemes are equally affected by this inaccuracy.

In Figs. 5(c) and (d), we demonstrate the root median square errors for the exact and approximate schemes, respectively, in the case where the sensors are not well calibrated. In particular, each sensor i experiences a gain g_i which is modeled as a uniform random variable in the interval $(0.8, 1.2)$. From these figures, we note that the examined gain perturbation leads to similar conclusions as the sensor locations perturbations.

6. Concluding remarks

In this work, the problem of localizing multiple acoustic sources using energy measurements from distributed sensors was considered. An alternating projection approach was formulated that exhibits lower computational complexity as compared to the maximum likelihood estimator. Furthermore, an efficient approximate scheme with lower computational complexity was proposed. Also, a computationally efficient initialization procedure was examined. Simulation results verified that the proposed localization method exhibits a performance close to the performance of the exact alternating projection algorithm, at a significantly lower computational complexity.

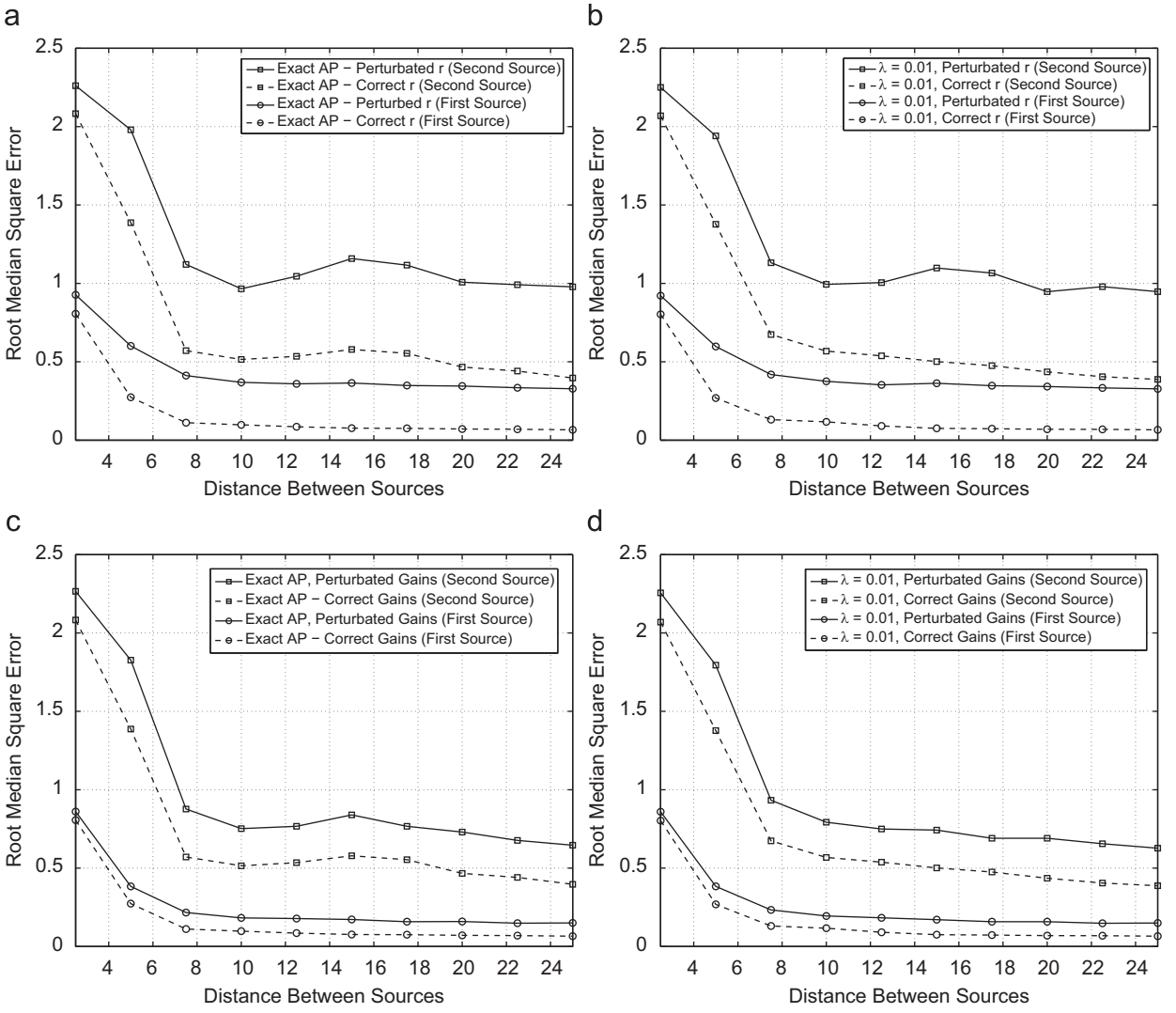

Fig. 5. (a) Root median square localization error under inaccurate sensor locations for the exact AP, (b) root median square localization error under inaccurate sensor locations for the low-complexity AP-based scheme, (c) root median square localization error under inaccurate sensor gains for the exact scheme, (d) root median square localization error under inaccurate sensor gains for the low-complexity AP-based scheme.

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